

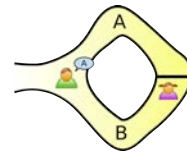
Type-checking Zero-knowledge

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Saarland University, Saarbrücken, Germany

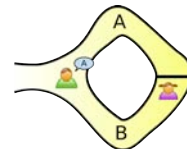
Joint work with: Michael Backes and Matteo Maffei

Zero-knowledge proofs



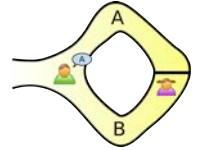
- ▶ Powerful cryptographic primitives
 - Prove the existence of an object with certain properties without revealing this object to anyone

Zero-knowledge proofs



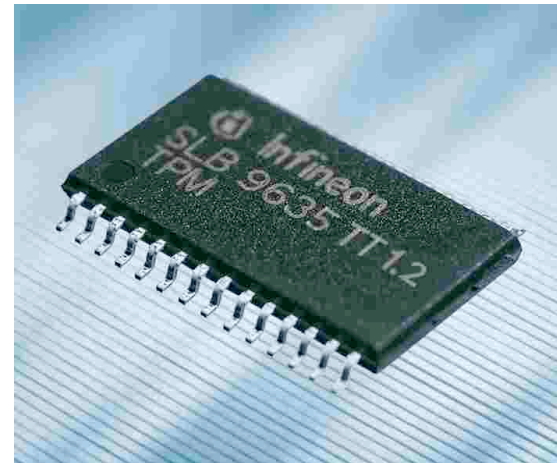
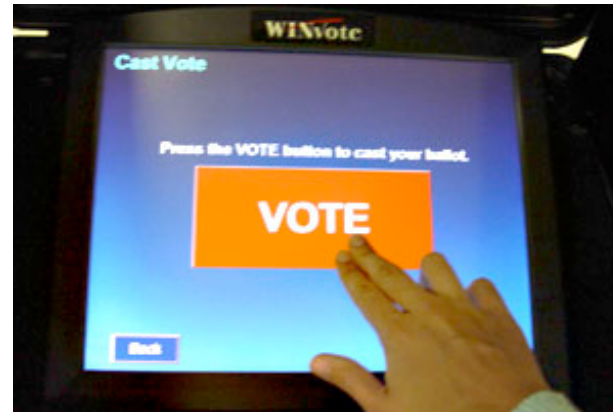
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 - Prove the existence of an object with certain properties without revealing this object to anyone
- ▶ Early constructions very general
 - But terribly inefficient
 - Very limited practical impact

Zero-knowledge proofs



- ▶ Powerful cryptographic primitives
 - Prove the existence of an object with certain properties without revealing this object to anyone
- ▶ Early constructions very general
 - But terribly inefficient
 - Very limited practical impact
- ▶ More recent research provided
 - Efficient constructions for special classes of statements
 - Constructions for non-interactive zero-knowledge

Many emerging applications use ZK



Lack of verification tools for ZK

- ▶ When we started this, there were no automated verification tools for protocols using zero-knowledge proofs as a primitive

Lack of verification tools for ZK

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- ▶ Security protocols are hard to get right

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- ▶ Security protocols are hard to get right
- ▶ Automated verification can really help protocol designers prevent high-level errors

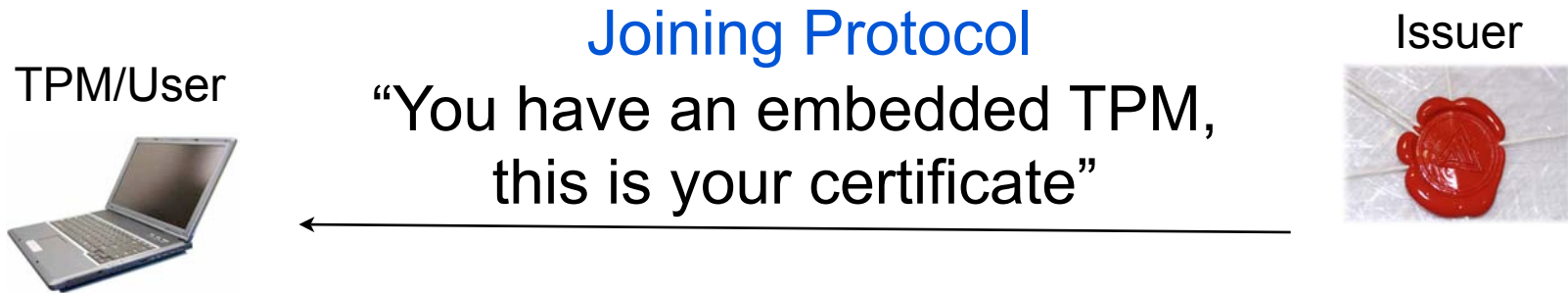
Lack of verification tools for ZK

- ▶ When we started this, there were no automated verification tools for protocols using zero-knowledge proofs as a primitive
- ▶ Security protocols are hard to get right
- ▶ Automated verification can really help protocol designers prevent high-level errors
- ▶ We provided two ways to automatically analyze protocols using zero-knowledge
 - Using ProVerif [Backes, Maffei & Unruh, S&P 2008]
 - Using a type system [Backes, Hrițcu & Maffei, CCS 2008]

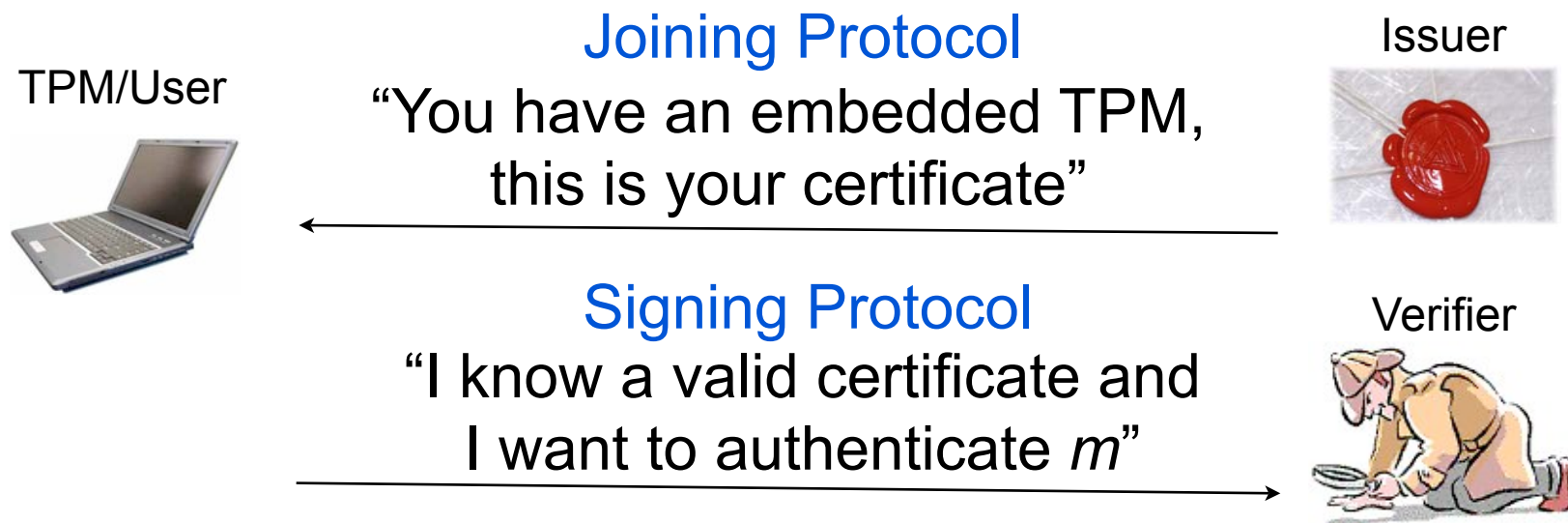
Outline

- ▶ Zero-knowledge proofs at work
 - *Direct Anonymous Attestation (DAA)* protocol (extremely simplified in my example)
- ▶ *Modeling zero-knowledge proofs symbolically*
- ▶ *Type system* to statically enforce authorization policies for protocols using zero-knowledge proofs
 - Extension of [Fournet, Gordon & Maffeis, CSF 2007]

Direct Anonymous Attestation (DAA)

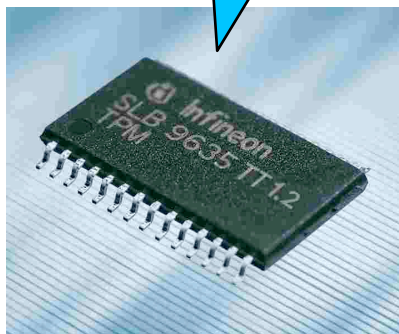


Direct Anonymous Attestation (DAA)



Direct Anonymous Attestation (DAA)

TPM/User



Joining Protocol

“You have an embedded TPM,
this is your certificate”



Issuer



Signing Protocol

“I know a valid certificate and
I want to authenticate m ”



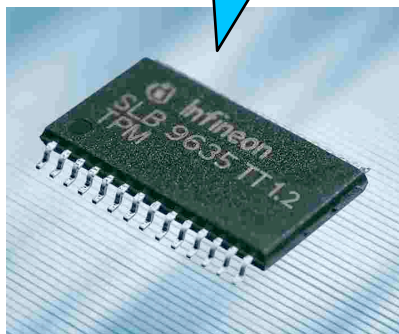
Verifier



The user proves that her platform has
a valid TPM inside (*attestation*)...

Direct Anonymous Attestation (DAA)

TPM/User



Joining Protocol

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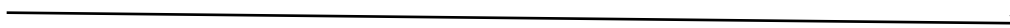


Issuer



Signing Protocol

“I know a valid certificate and
I want to authenticate m ”



Verifier



The user proves that her platform has
a valid TPM inside (*attestation*)...

... but the other parties do not learn *which*
TPM is used to authenticate m (*anonymity*)

Direct Anonymous Attestation (DAA)

TPM/User



f_{tpm}
(secret TPM identifier)

Issuer



Direct Anonymous Attestation (DAA)



Joining Protocol

The user receives a blind signature of f_{tpm} from the issuer

Direct Anonymous Attestation (DAA)

TPM/User



$\text{sign}(f_{\text{tpm}}, k_I)$

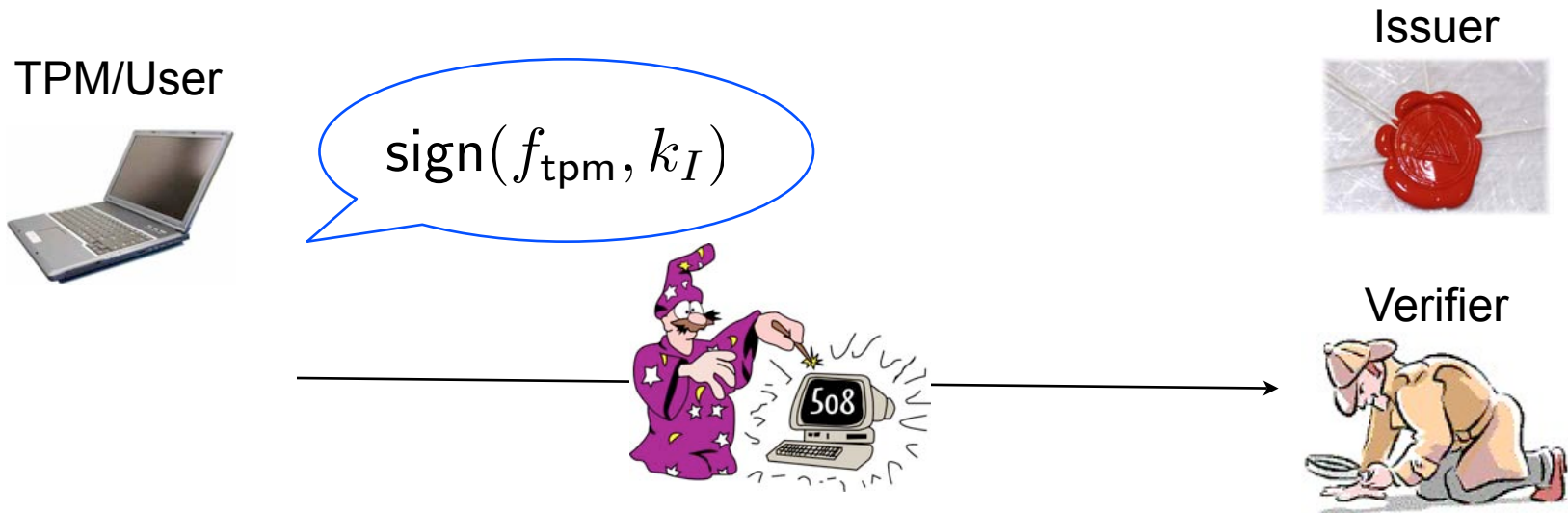
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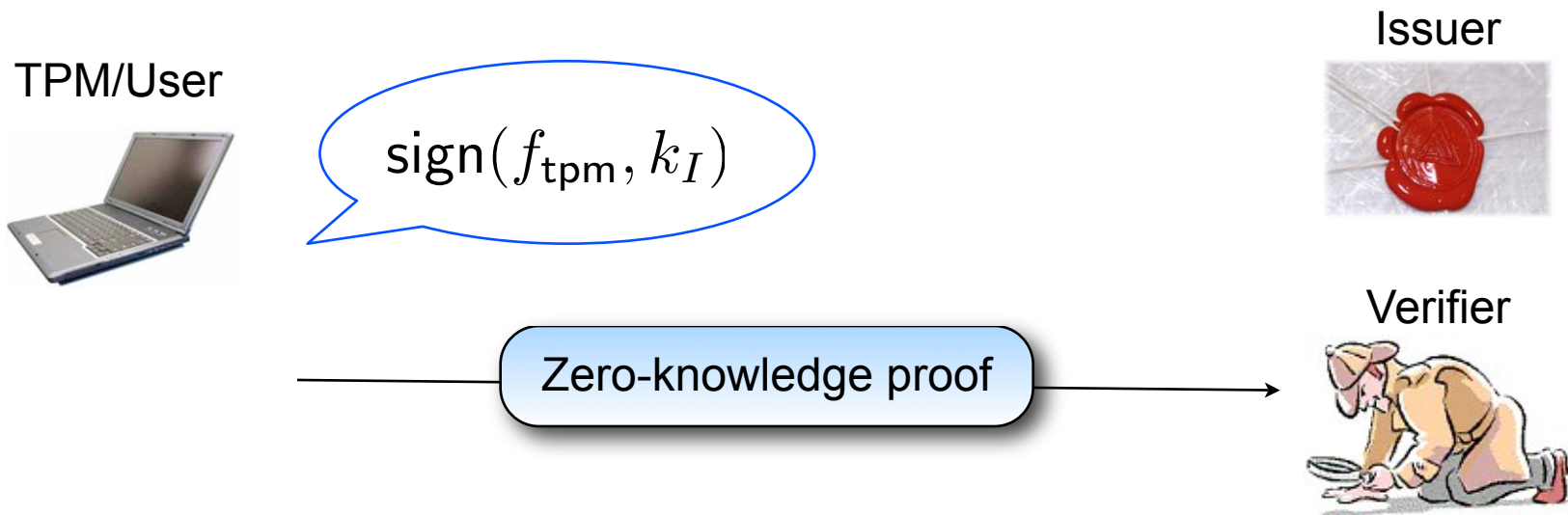
Direct Anonymous Attestation (DAA)



Signing Protocol

The user has to prove the knowledge of a certificate for the secret TPM identifier f_{tpm} ...
without revealing it!

Direct Anonymous Attestation (DAA)

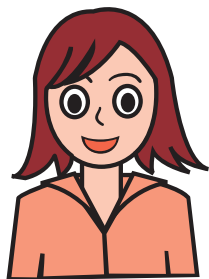


“there exists a secret α_f and a certificate α_{sign} such that the verification of α_{sign} with $\text{vk}(k_I)$ succeeds and the content of α_{sign} is α_f ”

Modeling zero-knowledge proofs symbolically

An extensible spi-calculus

Cryptographic primitives modeled as
user-defined constructors and *destructors*
[Abadi & Blanchet 2002]

 $\{M\}_k$ 

out(ch, enc(M , k)).

...

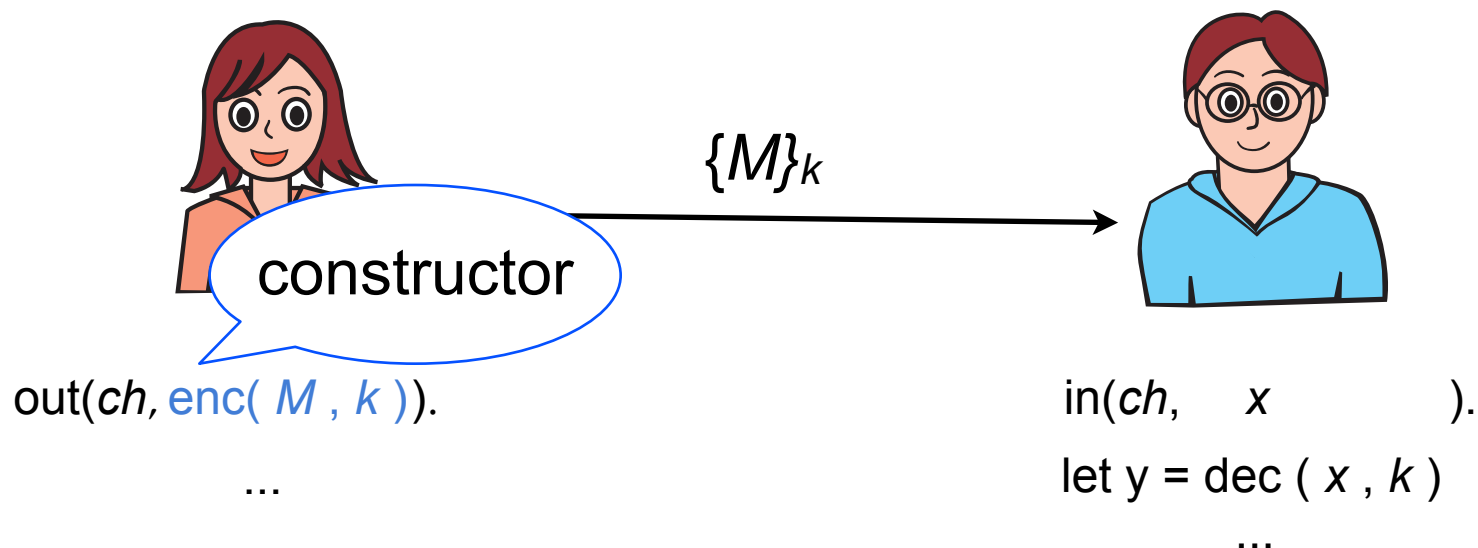
in(ch, x).

let y = dec (x , k)

...

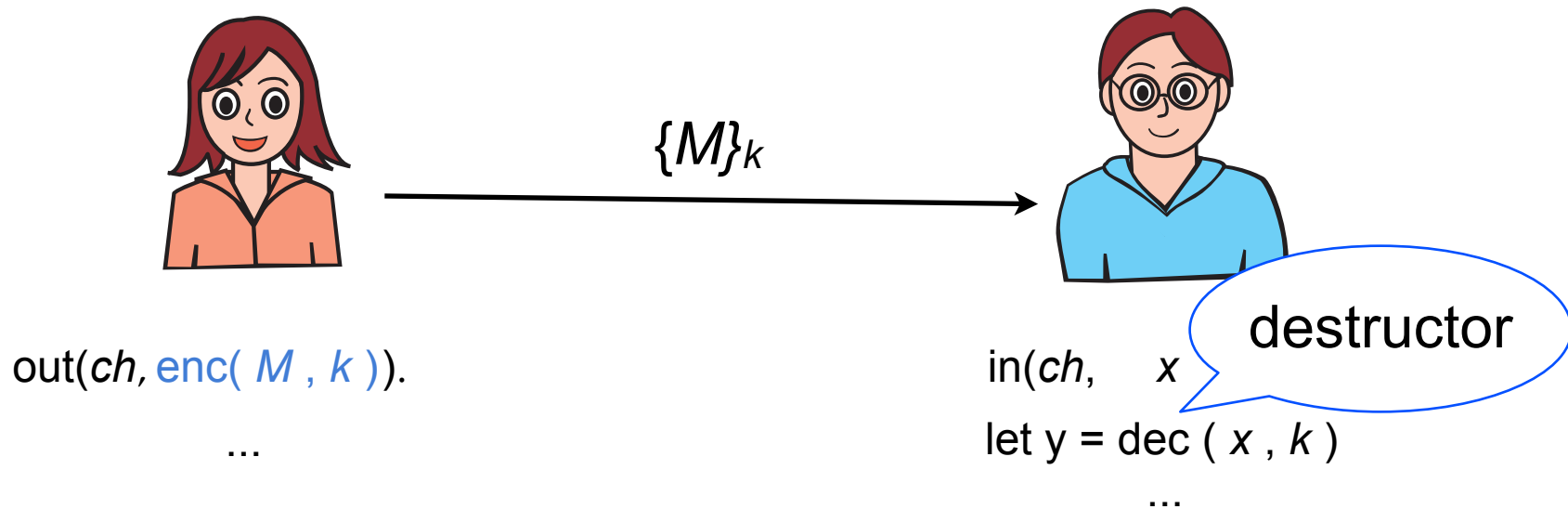
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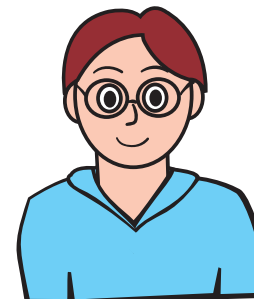
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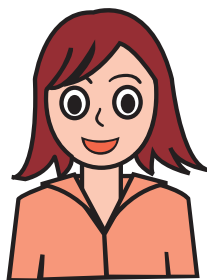
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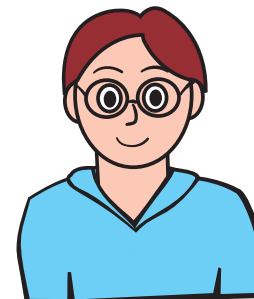
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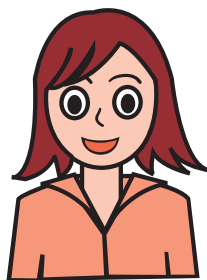
$\text{in}(ch, \text{enc}(M, k)).$

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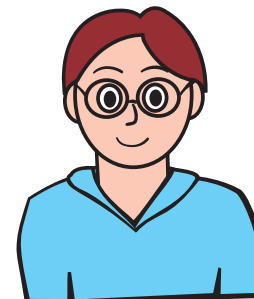
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let $y = \text{dec}(\text{enc}(M, k), k)$ then

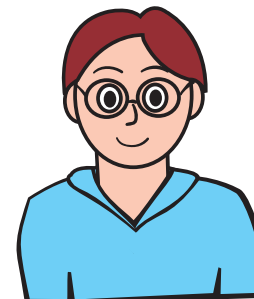
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$\{M\}_k$



let $y = \text{dec}(\text{enc}(M, k), k)$ then

Reduction relation \Downarrow

Reduction relation for destructors

The semantics of the calculus is parameterized by a user-defined reduction relation for destructors:

Crypto

$\text{dec}(\text{enc}(x,y), y) \Downarrow x$

$\text{chk}(\text{sign}(x,y), \text{vk}(y)) \Downarrow x$

Data

$\text{first}(\text{pair}(x,y)) \Downarrow x$

$\text{snd}(\text{pair}(x,y)) \Downarrow y$

$\text{eq}(x,x) \Downarrow \text{true}$

Logic

$\text{and}(\text{true}, \text{true}) \Downarrow \text{true}$

$\text{or}(x, \text{true}) \Downarrow \text{true}$

$\text{or}(\text{true}, x) \Downarrow \text{true}$

Abstraction of zero-knowledge

TPM/User



$\text{sign}(f_{\text{tpm}}, k_I)$

Zero-knowledge proof

Verifier



“there exists a secret α_f and a certificate α_{sign} such that the verification of α_{sign} with $\text{vk}(k_I)$ succeeds and the content α_{sign} of is α_f ”

Abstraction of zero-knowledge

TPM/User



$zk_{2,2,chk\#(\alpha_2,\beta_1)=\alpha_1}(f_{tpm}, \text{sign}(f_{tpm}, k_I); vk(k_I), m)$

Verifier



Abstraction of zero-knowledge

TPM/User



$$\text{zk}_{2,2,\text{chk}\#(\alpha_2,\beta_1)=\alpha_1}(f_{\text{tpm}}, \text{sign}(f_{\text{tpm}}, k_I); \text{vk}(k_I), m)$$

Verifier



$$\text{zk}_{2,2,\text{chk}\#(\alpha_2,\beta_1)=\alpha_1}(f_{\text{tpm}}, \text{sign}(f_{\text{tpm}}, k_I) ; \text{vk}(k_I), m)$$

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Verifier



private
messages

$$\text{zk}_{2,2,\text{chk}\#(\alpha_2,\beta_1)=\alpha_1}(f_{\text{tpm}}, \text{sign}(f_{\text{tpm}}, k_I); \text{vk}(k_I), m)$$

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Verifier



private
messages

public
messages

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Verifier



statement

private
messages

public
messages

$$\text{zk}_{2,2,\text{chk}^\#(\alpha_2,\beta_1)=\alpha_1}(f_{\text{tpm}}, \text{sign}(f_{\text{tpm}}, k_I); \text{vk}(k_I), m)$$

$$\text{chk}^\#(\alpha_2, \beta_1) = \alpha_1$$

Abstraction of zero-knowledge

TPM/User



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Verifier



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Abstraction of zero-knowledge

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Verifier



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Abstraction of zero-knowledge

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Verifier



statement

private
messages

public
messages

$$\text{zk}_{2,2,\text{chk}^\#(\alpha_2,\beta_1)=\alpha_1}(f_{\text{tpm}}, \text{sign}(f_{\text{tpm}}, k_I); \text{vk}(k_I), m)$$

$$\text{chk}^\#(\text{sign}(f_{\text{tpm}}, k_I), \text{vk}(k_I)) = f_{\text{tpm}}$$

DAA signing protocol (simplified)

TPM/User



$$\text{zk}_{2,2,\text{chk}^\#(\alpha_2,\beta_1)=\alpha_1}(f_{\text{tpm}}, \text{sign}(f_{\text{tpm}}, k_I); \text{vk}(k_I), m)$$

Verifier



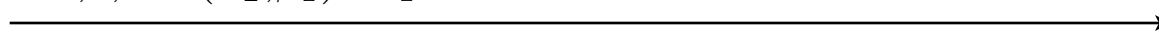
DAA = new k_I .
 new f_{tpm} .
 TPM | Verif | Issuer

TPM = new m .
 $\text{out}(c, \text{zk}_{2,2,\text{chk}^\#(\alpha_2,\beta_1)=\alpha_1}(f_{\text{tpm}}, \text{sign}(f_{\text{tpm}}, k_I); \text{vk}(k_I), m))$

DAA signing protocol (simplified)

TPM/User



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Verifier



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Verif = in(c, x).
 let $\langle x_m \rangle = \text{ver}_{2,2,\text{chk}^\#(\alpha_2,\beta_1)=\alpha_1}(x; \text{vk}(k_I))$ then

DAA signing protocol (simplified)

TPM/User



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 TPM | Verif | Issuer

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zero-knowledge
verification

Zero-knowledge verification

$$\text{ver}_{n,m,l,S}(\text{zk}_{n,m,S}(\tilde{N}; M_1, \dots, M_m), M_1, \dots, M_l) \Downarrow \langle M_{l+1}, \dots, M_m \rangle$$
$$\text{iff } S\{\tilde{N}/\tilde{\alpha}\}\{\tilde{M}/\tilde{\beta}\} \Downarrow_{\#} \text{true}$$

Zero-knowledge verification

$$\text{ver}_{n,m,l,S}(\text{zk}_{n,m,S}(\tilde{N}; M_1, \dots, M_m), M_1, \dots, M_l) \Downarrow \langle M_{l+1}, \dots, M_m \rangle$$
$$\text{iff } S\{\tilde{N}/\tilde{\alpha}\}\{\tilde{M}/\tilde{\beta}\} \Downarrow_{\#} \text{true}$$

Soundness and completeness:

Verification succeeds if and only if the proof is valid

Zero-knowledge:

Only the public messages can be extracted

For computational soundness see [Backes & Unruh, CSF 2008]

Type-checking zero-knowledge

Security annotations

DAA = new k_I .
 assume $\forall m. ((\exists x_f. \text{Send}(x_f, m) \wedge \text{OkTPM}(x_f)) \Rightarrow \text{Authenticate}(m))$ |
 new f_{tpm} .
 TPM | Verif | Issuer

TPM = new m .
 assume $\text{Send}(f_{\text{tpm}}, m)$ |
 out($c, \text{zk}_{2,2,\text{chk}^\#(\alpha_2,\beta_1)=\alpha_1}(f_{\text{tpm}}, \text{sign}(f_{\text{tpm}}, (k_I)); \text{vk}(k_I), m)$)

Verif = in(c, x).
 let $\langle x_m \rangle = \text{ver}_{2,2,\text{chk}^\#(\alpha_2,\beta_1)=\alpha_1}(x; \text{vk}(k_I))$ then
 assert $\text{Authenticate}(x_m)$

authorization policy
(OkTPM(x_f) assumed by the Issuer)

Security annotations

DAA = new k_I .
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 new f_{tpm} .
 TPM | Verif | Issuer

TPM = new m .
 assume $\text{Send}(f_{\text{tpm}}, m)$ |
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Verif = in(c, x).
 let $\langle x_m \rangle = \text{ver}_{2,2, \text{chk}^\#(\alpha_2, \beta_1) = \alpha_1}(x; \text{vk}(k_I))$ then
 assert $\text{Authenticate}(x_m)$

authorization policy
 (OkTPM(x_f) assumed by the Issuer)

Safety

A process is *safe* if each *assertion* is entailed
 at run-time by the current *assumptions*

Security annotations

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 assume $\forall m. ((\exists x_f. \text{Send}(x_f, m) \wedge \text{OkTPM}(x_f)) \Rightarrow \text{Authenticate}(m))$ |
 new f_{tpm} .
 TPM | Verif | Issuer

TPM = new m .
 assume $\text{Send}(f_{\text{tpm}}, m)$ |
 out($c, \text{zk}_{2,2, \text{chk}^\#(\alpha_2, \beta_1) = \alpha_1}(f_{\text{tpm}}, \text{sign}(f_{\text{tpm}}, (k_I)); \text{vk}(k_I), m)$)

Verif = in(c, x).
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 assert $\text{Authenticate}(x_m)$

authorization policy
(OkTPM(x_f) assumed by the Issuer)

Robust safety

A process is *robustly safe* if it is safe when run in parallel with an arbitrary opponent process.



Basic Types

new k_I .

assume $\forall m. ((\exists x_f. \text{Send}(x_f, m) \wedge \text{OkTPM}(x_f) \Rightarrow \text{Authenticate}(m))$ |

new f_{tpm} .

TPM | Verif | Issuer

TPM = new m : Un.

assume Send

out($c, \text{zk}_{2,2}, f_{\text{tpm}}, k_I; \text{vk}(k_I), m$)

Verif = in(c, x).

let $\langle x_m \rangle = \text{ver}_{2,2, \text{chk}^*(\alpha_2, \rho_1) = \langle \alpha_1 \rangle}(x; \text{vk}(k_I))$ then

assert $\text{Authenticate}(x_m)$

Type of
messages known to
the attacker

Basic Types

new k_I .

assume $\forall m. ((\exists x_f. \text{Send}(x_f, m) \wedge \text{OkTPM}(x_f) \Rightarrow \text{Authenticate}(m))$ |

new $f_{\text{tpm}}: \text{Private}$.

TPM | Verif |

TPM = new n

assume \exists

out($c, \text{zk}_{2,2,\text{chk}^\#(\alpha_2,\beta_1)=\alpha_1}(J_{\text{tpm}}, \text{sign}(f_{\text{tpm}}, k_I); \text{vk}(k_I), m)$)

Verif = in(c, x).

let $\langle x_m \rangle = \text{ver}_{2,2,\text{chk}^\#(\alpha_2,\beta_1)=\langle \alpha_1 \rangle}(x; \text{vk}(k_I))$ then

assert $\text{Authenticate}(x_m)$

Type of messages
unknown to the attacker

Basic Types

```

new  $k_I$ : SigKey( $\langle x_f : \text{Private} \rangle \{ \text{OkTPM}(x_f) \}$ )
assume  $\forall m. ((\exists x_f. \text{Send}(x_f, r) \wedge \text{OkTPM}(x_f) \wedge \text{Authenticate}(m)))$  |
new  $f_{\text{tpm}}$ : Private.
TPM | Verif | Issuer
TPM = new  $m$ : Un.
      assume Send( $f$ )
      out( $c, \text{zk}_{2,2,\text{chk}^\#(\alpha_2,r)}$ )
Verif = in( $c, x$ ).
        let  $\langle x_m \rangle = \text{ver}_{2,2,\text{chk}^\#(\alpha_2,r)}$ 
        assert Authenticate( $x_m$ )

```

Refinement type

[Bengtson et al., CSF 2008]

The key is used to sign only messages x_f of type Private such that $\text{OkTPM}(x_f)$ is entailed by the current assumptions

Basic Types

```

new  $k_I$ : SigKey( $\langle x_f : \text{Private} \rangle \{ \text{OkTPM}(x_f) \}$ )
assume  $\forall m. ((\exists x_f. \text{Send}(x_f, m) \wedge \text{OkTPM}(x_f)) \Rightarrow \text{Authenticate}(m))$  |
new  $f_{\text{tpm}}$ : Private.
TPM | Verif | Issuer

TPM = new  $m$ : Un.
      assume Send( $f_{\text{tpm}}, m$ ) |
      out( $c, \text{zk}_{2,2,\text{chk}^\#(\alpha_2,\beta_1)=\alpha_1}(f_{\text{tpm}}, \text{sign}(f_{\text{tpm}}, k_I); \text{vk}(k_I), m)$ )

Verif = in( $c, x$ ).
        let  $\langle x_m \rangle = \text{ver}_{2,2,\text{chk}^\#(\alpha_2,\beta_1)=\langle \alpha_1 \rangle}(x; \text{vk}(k_I))$  then
        assert Authenticate( $x_m$ )

```

The user knows that $\text{Send}(f_{\text{tpm}}, m)$ (*local assumption*) and $\text{OkTPM}(f_{\text{tpm}})$ (*signature check*) are entailed...
but the verifier doesn't!

Basic Types

```

new  $k_I$ : SigKey( $\langle x_f : \text{Private} \rangle \{ \text{OkTPM}(x_f) \}$ )
assume  $\forall m. ((\exists x_f. \text{Send}(x_f, m) \wedge \text{OkTPM}(x_f)) \Rightarrow \text{Authenticate}(m))$  |
new  $f_{\text{tpm}}$ : Private.
TPM | Verif | Issuer

TPM = new  $m$ : Un.
      assume Send( $f_{\text{tpm}}, m$ ) |
      out( $c, \text{zk}_{2,2,\text{chk}^\#(\alpha_2,\beta_1)=\alpha_1}(f_{\text{tpm}}, \text{sign}(f_{\text{tpm}}, k_I); \text{vk}(k_I), m)$ )

Verif = in( $c, x$ ).
        let  $\langle x_m \rangle = \text{ver}_{2,2,\text{chk}^\#(\alpha_2,\beta_1)=\langle \alpha_1 \rangle}(x; \text{vk}(k_I))$  then
        assert Authenticate( $x_m$ )

```

How can we statically transfer these predicates from the user to the verifier?

Typing zero-knowledge proofs

TPM/User



$$\text{zk}_{2,2,\text{chk}\#(\alpha_2,\beta_1)=\langle\alpha_1\rangle}(f_{\text{tpm}}, \text{sign}(f_{\text{tpm}}, k_I); \text{vk}(k_I), m)$$

Verifier



As usual! Use a refinement type for the key and ...

Typing zero-knowledge proofs

TPM/User



$$\text{zk}_{2,2,\text{chk}\#}(\alpha_2, \beta_1) = \langle \alpha_1 \rangle (f_{\text{tpm}}, \text{sign}(f_{\text{tpm}}, k_I); \text{vk}(k_I), m)$$

Verifier



Technical issue

Zero-knowledge proofs don't necessarily rely on keys...

Typing zero-knowledge proofs

TPM/User



$$\text{zk}_{2,2,\text{chk}^\#(\alpha_2,\beta_1)=\langle\alpha_1\rangle}(f_{\text{tpm}}, \text{sign}(f_{\text{tpm}}, k_I); \text{vk}(k_I), m)$$

Verifier



$$\text{ZK}_{2,2,\text{chk}^\#(\alpha_2,\beta_1)=\langle\alpha_1\rangle} \left(\begin{array}{l} \langle y_k : \text{VerKey}(\langle x : \text{Private} \rangle \{ \text{OkTPM}(x) \}), y_m : \text{Un} \rangle \\ \{ \exists x_f, x_s. \text{Send}(x_f, y_m) \wedge \text{OkTPM}(x_f) \} \end{array} \right)$$

For each statement in the protocol the user needs to annotate such a type

Typing zero-knowledge proofs

TPM/User



Verifier



$$\text{zk}_{2,2,\text{chk}^\#(\alpha_2,\beta_1)=\langle\alpha_1\rangle}(f_{\text{tpm}}, \text{sign}(f_{\text{tpm}}, k_I); \text{vk}(k_I), m)$$

Type of public messages

$$\text{ZK}_{2,2,\text{chk}^\#(\alpha_2,\beta_1)=\langle\alpha_1\rangle} \left(\langle y_k : \text{VerKey}(\langle x : \text{Private} \rangle \{ \text{OkTPM}(x) \}), y_m : \text{Un} \rangle \{ \exists x_f, x_s. \text{Send}(x_f, y_m) \wedge \text{OkTPM}(x_f) \} \right)$$

Typing zero-knowledge proofs

TPM/User



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$$\text{zk}_{2,2,\text{chk}^\#(\alpha_2,\beta_1)=\langle\alpha_1\rangle}(f_{\text{tpm}}, \text{sign}(f_{\text{tpm}}, k_I); \text{vk}(k_I), m)$$

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Formula entailed by the current assumptions (private messages existentially quantified)

Type-checking the prover

$$\text{ZK}_{2,2,\text{chk}^\#(\alpha_2,\beta_1)=\langle\alpha_1\rangle} \left(\begin{array}{l} \langle y_k : \text{VerKey}(\langle x : \text{Private} \rangle \{ \text{OkTPM}(x) \}), y_m : \text{Un} \rangle \\ \{ \exists x_f, x_s. \text{Send}(x_f, y_m) \wedge \text{OkTPM}(x_f) \} \end{array} \right)$$

$$\begin{aligned} \Gamma = & \dots \\ & k_I : \text{SigKey}(\langle x_f : \text{Private} \rangle \{ \text{OkTPM}(x_f) \}), \\ & f_{\text{tpm}} : \text{Private}, \\ & m : \text{Un}, \\ & \forall m. ((\exists x_f. \text{Send}(x_f, m) \wedge \text{OkTPM}(x_f)) \Rightarrow \text{Authenticate}(m)), \\ & \text{OkTPM}(f_{\text{tpm}}), \\ & \text{Send}(f_{\text{tpm}}, m) \end{aligned}$$



- Type of public messages
- Logical formula entailed

$$\text{TPM} = \dots \text{out}(c, \text{zk}_{2,2,\text{chk}^\#(\alpha_2,\beta_1)=\langle\alpha_1\rangle}(f_{\text{tpm}}, \text{sign}(f_{\text{tpm}}, k_I); \text{vk}(k_I), m))$$

Type-checking the verifier

$$\text{ZK}_{2,2,\text{chk}^\#(\alpha_2,\beta_1)=\langle\alpha_1\rangle} \left(\begin{array}{l} \langle y_k : \text{VerKey}(\langle x : \text{Private} \rangle \{ \text{OkTPM}(x) \}), y_m : \text{Un} \rangle \\ \{ \exists x_f, x_s. \text{Send}(x_f, y_m) \wedge \text{OkTPM}(x_f) \} \end{array} \right)$$

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$$\begin{aligned} \text{Verif} = & \text{in}(c, x). \\ & \text{let } \langle y_m \rangle = \text{ver}_{2,2,\text{chk}^\#(\alpha_2,\beta_1)=\langle\alpha_1\rangle}(x; \text{vk}(k_I)) \text{ then} \\ & \text{assert } \text{Authenticate}(y_m) \end{aligned}$$

Type-checking the verifier

$$\text{ZK}_{2,2,\text{chk}^\#(\alpha_2,\beta_1)=\langle\alpha_1\rangle} \left(\begin{array}{l} \langle y_k : \text{VerKey}(\langle x : \text{Private} \rangle \{ \text{OkTPM}(x) \}), y_m : \text{Un} \rangle \\ \{ \exists x_f, x_s. \text{Send}(x_f, y_m) \wedge \text{OkTPM}(x_f) \} \end{array} \right)$$

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If verification succeeds, can we give $\langle y_m \rangle$ type $\langle y_m : \text{Un} \rangle \{ \exists x_f, x_s. \text{Send}(x_f, y_m) \wedge \text{OkTPM}(x_f) \}$?

$$\begin{aligned} \text{Verif} = & \text{in}(c, x). \\ & \text{let } \langle y_m \rangle = \text{ver}_{2,2,\text{chk}^\#(\alpha_2,\beta_1)=\langle\alpha_1\rangle}(x; \text{vk}(k_I)) \text{ then} \\ & \text{assert } \text{Authenticate}(y_m) \end{aligned}$$

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$$\Gamma = \dots$$

$$k_I : \text{SigKey}(\langle x_f : \text{Private} \rangle \{ \text{OkTPM}(x_f) \}),$$

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In general not!

$$\text{Verif} = \text{in}(c, x).$$

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Type-checking the verifier

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$\Gamma = \dots$

$k_I : \text{SigKey}(\langle x_f : \text{Private} \rangle \{ \text{OkTPM}(x_f) \})$

$f_{\text{tpm}} : \text{Private},$

$\forall m. ((\exists x_f. \text{Send}(x_f, m) \wedge \text{OkTPM}(x_f)) \rightarrow \text{OkTPM}(m)),$

Does the zero-knowledge proof
come from the adversary or
from an honest participant?



If verification succeeds, can we give $\langle y_m \rangle$ type
 $\langle y_m : \text{Un} \rangle \{ \exists x_f, x_s. \text{Send}(x_f, y_m) \wedge \text{OkTPM}(x_f) \}$?

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assert $\text{Authenticate}(y_m)$

Type-checking the verifier

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Conceptual issue

We do not know whether the zero-knowledge proof comes from an honest participant or from the adversary!

Zero-knowledge verification

$$\text{ZK}_{2,2,\text{chk}^\#(\alpha_2,\beta_1)=\langle\alpha_1\rangle} \left(\begin{array}{l} \langle y_k : \text{VerKey}(\langle x : \text{Private} \rangle \{ \text{OkTPM}(x) \}), y_m : \text{Un} \rangle \\ \{ \exists x_f, x_s. \text{Send}(x_f, y_m) \wedge \text{OkTPM}(x_f) \} \end{array} \right)$$

Statement

$$\text{chk}^\#(x_s, \text{vk}(k_I)) = x_f$$

Typing environment

$$\text{vk}(k_I) : \text{VerKey}(\langle x : \text{Private} \rangle \{ \text{OkTPM}(x) \})$$



Take the statement
(instantiated with the public
messages you know) and the
typing environment

Zero-knowledge verification

$$\text{ZK}_{2,2,\text{chk}^\#(\alpha_2,\beta_1)=\langle\alpha_1\rangle} \left(\begin{array}{l} \langle y_k : \text{VerKey}(\langle x : \text{Private} \rangle \{ \text{OkTPM}(x) \}), y_m : \text{Un} \rangle \\ \{ \exists x_f, x_s. \text{Send}(x_f, y_m) \wedge \text{OkTPM}(x_f) \} \end{array} \right)$$

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$$\dots, x_f : \text{Private}, \text{OkTPM}(x_f)$$



The type of the signing key gives us the type of the first private message (existentially quantified)!

Zero-knowledge verification

$$\text{ZK}_{2,2,\text{chk}^\#(\alpha_2,\beta_1)=\langle\alpha_1\rangle} \left(\begin{array}{l} \langle y_k : \text{VerKey}(\langle x : \text{Private} \rangle \{ \text{OkTPM}(x) \}), y_m : \text{Un} \rangle \\ \{ \exists x_f, x_s. \text{Send}(x_f, y_m) \wedge \text{OkTPM}(x_f) \} \end{array} \right)$$

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$$\text{vk}(k_I) : \text{VerKey}(\langle x : \text{Private} \rangle \{ \text{OkTPM}(x) \})$$



$$\dots, x_f : \text{Private}, \text{OkTPM}(x_f)$$



The prover is honest,
since she knows a message
of type Private!

Zero-knowledge verification

$$\text{ZK}_{2,2,\text{chk}^\#(\alpha_2,\beta_1)=\langle\alpha_1\rangle} \left(\begin{array}{l} \langle y_k : \text{VerKey}(\langle x : \text{Private} \rangle \{ \text{OkTPM}(x) \}), y_m : \text{Un} \rangle \\ \{ \exists x_f, x_s. \text{Send}(x_f, y_m) \wedge \text{OkTPM}(x_f) \} \end{array} \right)$$

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Typing environment

$$\text{vk}(k_I) : \text{VerKey}(\langle x : \text{Private} \rangle \{ \text{OkTPM}(x) \})$$



We can now exploit the type of the zero-knowledge proof!

↓

..., $x_f : \text{Private}$,
 $\text{OkTPM}(x_f)$

↓

..., $x_f : \text{Private}$, $y_m : \text{Un}$
 $\text{OkTPM}(x_f)$, $\text{Send}(x_f, y_m)$

Zero-knowledge verification

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$$y_m : \text{Un}.$$

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$$\dots, x_f : \text{Private},$$

$$\text{OkTPM}(x_f)$$

$$\dots, x_f : \text{Private}, y_m : \text{Un}$$

$$\text{OkTPM}(x_f), \text{Send}(x_f, y_m)$$

Type-checking the verifier

$$\begin{aligned}\Gamma = & \dots \\ & k_I : \text{SigKey}(\langle x_f : \text{Private} \rangle \{ \text{OkTPM}(x_f) \}), \\ & f_{\text{tpm}} : \text{Private}, \\ & \forall m. ((\exists x_f. \text{Send}(x_f, m) \wedge \text{OkTPM}(x_f) \Rightarrow \text{Authenticate}(m)), \\ & y_m : \text{Un}, \\ & \exists x_f. \text{Send}(x_f, y_m) \wedge \text{OkTPM}(x_f),\end{aligned}$$
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$$\text{Verif} = \dots$$

$$\text{assert } \text{Authenticate}(y_m)$$

Theorem (Robust safety)

If $\Gamma \vdash P$, then P is robustly safe

Typed analysis of zero-knowledge

- ▶ Fully automated (we implemented a type-checker and use SPASS to discharge FOL proof obligations)
- ▶ Efficient (analysis of DAA takes less than 3s)
- ▶ Compositional and therefore scalable
- ▶ Predictable termination behavior
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This code is safe

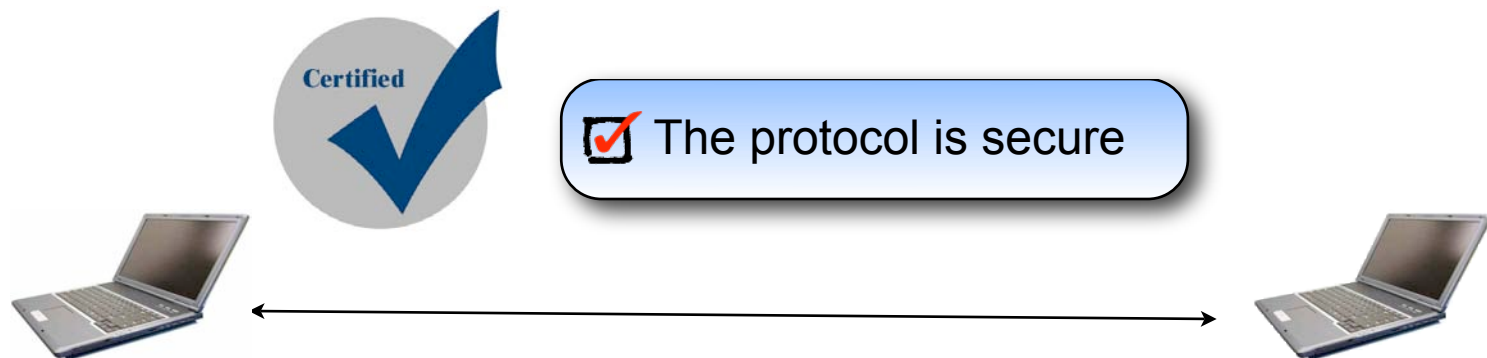


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Take home

- ▶ Zero-knowledge proofs are given *refinement types* where the private messages are *existentially quantified*
- ▶ The *prover* asserts *only valid statements*
- ▶ The *verifier* can assume the formula in the type if
 - the formula is entirely derived from the zero-knowledge statement (often too weak)
 - the proof comes from an *honest party* (*statically checked* by looking at the statement and at the type of the matched public messages)



Future work



- ▶ *Sound implementation* of our abstraction
 - Identified assumptions for computational soundness in [Backes & Unruh, CSF 2008]

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THANK YOU!

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