Type-checking Zero-knowledge

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Abstract

This paper presents the first type system for statically analyzing security protocols that are based on zero-knowledge proofs. We show how certain properties offered by zero-knowledge proofs can be characterized in terms of authorization policies and statically enforced by a type system. The analysis is modular and compositional, and provides security proofs for an unbounded number of protocol executions. We develop a new type-checker that conducts the analysis in a fully automated manner. We exemplify the applicability of our technique to real-world protocols by verifying the authenticity and secrecy properties of the Direct Anonymous Attestation (DAA) protocol. The analysis of DAA takes less than three seconds.

1   Introduction

The design of cryptographic protocols is notoriously difficult and error-prone, and manual security proofs for such protocols are difficult to do. The multitude of attacks on existing cryptographic protocols reported lately (e.g., [37, 15, 25, 18]) demonstrates the need for formalizing the intended security properties and developing automated techniques for automatically verifying these properties. Logic-based authorization policies constitute a well-established and expressive framework for describing a wide range of security properties of cryptographic protocols, varying from authenticity properties to access control policies [2]. Type systems are particularly salient tools to statically and automatically enforce authorization policies on abstract protocol specifications [26, 27] and on concrete protocol implementations [13]. Type systems require little human effort and provide security proofs for an unbounded number of protocol executions. Furthermore, the analysis is modular, compositional, and usually guaranteed to terminate.

One of the central challenges in the verification of authorization policies for modern applications is the expressiveness of the analysis and its ability to statically characterize the security properties guaranteed by complex cryptographic operations. For instance, current analysis techniques support traditional cryptographic primitives such as encryption and digital signatures, but until recently [11] they could not cope with the most prominent and innovative modern cryptographic primitive: zero-knowledge proofs [28].

A zero-knowledge proof combines two seemingly contradictory properties. First, it constitutes a proof of a statement that cannot be forged, i.e., it is impossible, or at least computationally infeasible, to produce a zero-knowledge proof of a wrong statement. Second, a zero-knowledge proof does not reveal any information besides the bare fact that the statement is valid. Early general-purpose zero-knowledge proofs were primarily designed for showing the existence of such proofs for the class of statements under consideration. These proofs were very inefficient and consequently of only limited use in practical applications. The recent advent of efficient zero-knowledge proofs for special classes of statements is rapidly changing this scenario. The unique

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security features that zero-knowledge proofs offer combined with the possibility to efficiently implement some of these proofs non-interactively have paved the way for their deployment in modern cryptographic applications. In fact, many anonymity protocols [16, 36] and electronic voting protocols [34, 6, 20] heavily rely on zero-knowledge proofs. These zero-knowledge proofs provide security properties that go far beyond the traditional and well-understood secrecy and authenticity properties. For instance, zero-knowledge proofs may guarantee authentication yet preserve the anonymity of protocol participants, as in the Pseudo Trust protocol [36], or they may prove the reception of a certificate from a trusted server without revealing the actual content, as in the Direct Anonymous Attestation (DAA) protocol [16].

Statically analyzing protocols that use zero-knowledge proofs is conceptually and technically challenging. While the existing techniques for type-checking cryptographic protocols usually rely on the type of keys for typing cryptographic messages, these techniques do not directly apply to zero-knowledge proofs, since in general zero-knowledge proofs do not depend on any key infrastructure.

1.1 Our Contributions

This paper presents the first type system for statically analyzing the security of protocols based on non-interactive zero-knowledge proofs. We show how the safety properties guaranteed by zero-knowledge proofs can be formulated in terms of authorization policies and statically enforced by a type system. Our approach extends the state-of-the-art type system for authorization policies proposed by Fournet et al. [27]. Zero-knowledge proofs are given dependent types where the messages kept secret by the proof are existentially quantified in the logic. The fundamental idea is to express zero-knowledge statements as logical formulas and to define the type of zero-knowledge proofs using these formulas. The user still has the possibility to extend such types with additional logical formulas describing protocol-dependent security properties.

We develop a new type-checker that automates the analysis. The tool verifies that protocol specifications are well-typed and relies on the first-order logic automated theorem prover SPASS [38] to discharge proof obligations. The analysis is modular and compositional, and provides security proofs for an unbounded number of protocol executions.

We exemplify the applicability of our technique to real-world protocols by verifying the Direct Anonymous Attestation protocol (DAA) [16]. We formalize the authenticity properties of this protocol in terms of authorization policies and we apply our type system to statically verify them. Our type-checker analyzed this sophisticated protocol in less than three seconds. This promising result indicates that our static analysis technique has the potential to scale up to industrial-size protocols.

1.2 Related Work

Dating back to the seminal work by Abadi on secrecy by typing [1, 3], type systems were successfully used to analyze a wide range of security properties of cryptographic protocols, ranging from authenticity properties [30, 32, 33, 17, 8], to security despite compromised participants [31, 17, 27, 21], to authorization policies [26, 27, 13]. As mentioned before, type-checking is efficient, modular, compositional, and usually guaranteed to terminate. None of the existing type systems is, however, capable of dealing with zero-knowledge proofs.

To the best of our knowledge, ProVerif [14, 5] is the only automatic tool that has been applied to the analysis of protocols that use zero-knowledge proofs [11, 9, 23]. This tool is based on Horn-clause resolution and can analyze trace-based security properties as well as behavioral properties. The analysis with ProVerif is not compositional and often has unpredictable termination behaviour, with seemingly harmless code changes leading to divergence. Also, as

argued in [13], type systems scale better to large protocols and more efficiently analyze protocol implementations. In terms of expressiveness, ProVerif can deal with behavioral properties that are generally out of scope for current type systems (e.g., privacy and coercion-resistance in electronic-voting protocols [22, 9]), but is restricted to cryptographic primitives that can be expressed as convergent or linear equational theories [5]. Our analysis does not pose any constraint on the semantics of cryptographic primitives and, as opposed to ProVerif, can deal with authorization policies using arbitrary logical structure (e.g., arbitrarily nested quantifiers).

1.3 Outline

Section 2 illustrates our approach on a simple protocol for anonymous trust. Section 3 describes the process calculus we use to model security protocols that use zero-knowledge proofs. Section 4 presents our type system for zero-knowledge. Section 5 discusses the implementation of our type-checker and the experimental evaluation of our technique. In Section 6 we apply our type system to analyze the Direct Anonymous Attestation (DAA) protocol. In Section 7 we explain how to use zero-knowledge proofs in the design of systems that guarantee security despite partial compromise. Section 8 concludes and provides directions for future work. Appendix A describes the syntax and semantics of the applied pi-calculus with constructors and destructors. Appendix B lists all the typing rules of our type system. Appendix C is devoted to proving that our type system is sound. Appendix D presents the extension of our technique to blind signatures, which was necessary for modeling and analyzing DAA.

2 Illustrative Example

This section introduces the types of zero-knowledge proofs and highlights the fundamental ideas of our type system, which will be elaborated in more detail in the following sections. As a running example, we consider a simple protocol for anonymous trust that is inspired by the Pseudo Trust protocol proposed by Lu et al. in [36]. The goal of this protocol is to allow parties to exchange data proving each other's trust level while preserving anonymity. These two seemingly conflicting requirements are met by an authentication scheme based on zero-knowledge proofs. We show how to characterize the notion of anonymous trust in terms of authorization policies and how to statically verify them by our type system.

2.1 A Protocol for Anonymous Trust

Each party has a public pseudonym, which is the hash of a secret m_s. This pseudonym replaces the actual identity of the party in the protocol. An arbitrary trust-management system like EigenTrust [35] or XenoTrust [24] can be used to certify the trust level of each pseudonym. Whenever a party (*prover*) wants to send a message m_p to another party (*verifier*), she has to bind m_p to her own pseudonym $\mathsf{hash}(m_s)$ and, in order to avoid impersonation, she has to prove the knowledge of m_s without revealing it. This authentication scheme is realized by a non-interactive zero-knowledge proof that is sent from the prover to the verifier. The zero-knowledge proof guarantees that the prover knows m_s and additionally provides the non-malleability of m_p, i.e., changing m_p requires the adversary to redo the proof and thus to know m_s. The goal of this protocol is allowing the verifier to associate the trust level of the prover to m_p.

2.2 Zero-knowledge Proofs and Authorization Policies

Following [11], we represent our zero-knowledge proof by the following applied pi-calculus term: $\mathsf{zk}_{1,2,\beta_1=\mathsf{hash}(\alpha_1)}(m_s; \mathsf{hash}(m_s), m_p)$. In our setting, a zero-knowledge proof

$\mathsf{zk}_{n,m,S}(N_1, \ldots, N_n; M_1, \ldots, M_m)$ has $n + m$ arguments. The first n arguments $N_1, \ldots, N_n$ form the *private component* of the proof and are kept secret (m_s in the example), while the other m arguments $M_1, \ldots, M_m$ form the *public component* and are revealed to the verifier ($\mathsf{hash}(m_s)$ and m_p in the example). The *statement S* of the zero-knowledge proof is a Boolean formula over the placeholders α_i and β_j (with $i \in [1, n]$ and $j \in [1, m]$), which stand for the argument N_i in the private component and the argument M_j in the public component, respectively. The verification of a zero-knowledge proof succeeds if and only if the statement obtained by replacing the place-holders by the corresponding private and public arguments holds true.

In order to express the security property guaranteed by this protocol as an authorization policy, we decorate the security-related protocol events as follows:

assume $\mathsf{Authenticate}(m_p, m_s)$ $\qquad\qquad\qquad\qquad\qquad$ assume $\mathsf{Trust}(pseudo, k)$

$$P \xrightarrow{\quad\mathsf{zk}_{1,2,\beta_1=\mathsf{hash}(\alpha_1)}(m_s;\mathsf{hash}(m_s),m_p)\quad} V$$

assert $\mathsf{Associate}(m_p, k)$

Before generating the zero-knowledge proof to authenticate m_p using the knowledge of the secret m_s, the prover P assumes the logical predicate $\mathsf{Authenticate}(m_p, m_s)$. Before receiving the zero-knowledge proof, the verifier V knows that the trust level associated to the pseudonym *pseudo* is k and assumes the predicate $\mathsf{Trust}(pseudo, k)$. As discussed before, this information can be obtained by means of an external trust-management system. The verifier receives and verifies the zero-knowledge proof, checking that the first argument in the public component is *pseudo*. This guarantees that the prover knows the secret m_s such that $pseudo = \mathsf{hash}(m_s)$ and allows the verifier to assert that the formula $\mathsf{Associate}(m_p, k)$ holds. The authorization policy can be expressed as follows:

$$Policy := \forall m_p, k, m_s.(\mathsf{Authenticate}(m_p, m_s) \wedge \mathsf{Trust}(\mathsf{hash}(m_s), k)) \Rightarrow \mathsf{Associate}(m_p, k)$$

Since this is the only assumption where the $\mathsf{Associate}$ predicate occurs, the verifier is allowed to give m_p trust level k only if the prover wants to authenticate m_p and knows some m_s for which the trust level of the pseudonym $\mathsf{hash}(m_s)$ is k. Everything that is not explicitly allowed by the authorization policy is prohibited.

2.3 The Type of Zero-knowledge Proofs

To illustrate our technique, let us consider the type associated to the zero-knowledge proof $\mathsf{zk}_{1,2,\beta_1=\mathsf{hash}(\alpha_1)}(m_s; \mathsf{hash}(m_s), m_p)$:

$$\mathsf{ZKProof}_{1,2,\beta_1=\mathsf{hash}(\alpha_1)}(\langle y_1 : \mathsf{Hash}(\mathsf{Private}), y_2 : \mathsf{Un}\rangle\{\exists x.\ y_1 = \mathsf{hash}(x) \wedge \mathsf{Authenticate}(y_2, x)\})$$

This dependent type indicates that the public component is composed of two messages. The first message y_1 is of type $\mathsf{Hash}(\mathsf{Private})$, i.e., it is the hash of a secret message. The type $\mathsf{Private}$ describes messages that are not known to the adversary. The second message y_2 is of type Un (untrusted), i.e., it may come from and be sent to the adversary.

The logical formula $\exists x.\ y_1 = \mathsf{hash}(x) \wedge \mathsf{Authenticate}(y_2, x)$ says that y_1 is the hash of some secret x such that $\mathsf{Authenticate}(y_2, x)$ has been assumed by the prover. Note that this formula contains an equality constraint on the structure of messages as well as a logical predicate.

After the verification of the zero-knowledge proof, the verifier can safely assume that the formula holds true. The constraint $\exists x.\ y_1 = \mathsf{hash}(x)$ is guaranteed by the semantics of the zero-knowledge proof, while the assumption $\mathsf{Authenticate}(y_2, x)$ is enforced by our

type system. The verifier can thus logically derive the following formula: $\exists m_s.\ pseudo = $ hash$(m_s) \wedge$ Authenticate$(m_p, m_s) \wedge$ Trust$(pseudo, k)$. By a standard logical property that allows replacing equals by equals, the authorization policy allows the verifier to derive Associate(m_p, k).

3   Calculus

We consider a variant of the applied pi-calculus with constructors and destructors similar to the one in [4], and we extend it with zero-knowledge proofs. Following [27, 13], the calculus also includes special operators to assume and assert logical formulas. This section overviews the syntax and semantics of the calculus. More details are given in Appendix A.

3.1   Constructors and Terms

Constructors are function symbols that are used to build terms. The set of constructors includes pk that yields the public encryption key corresponding to a decryption key; enc for public-key encryption; vk that yields the verification key corresponding to a signing key; sign for digital signatures; and hash for hashes. The constant true represents the respective Boolean value, and has its canonical meaning in the authorization logic.

The set of *terms* (ranged over by K, L, M and N) is the free algebra built from names (a, b, c, m, n, and k), variables (x, y, z, v, and w), tuples ($\langle M_1, \ldots, M_n \rangle$), and constructors applied to other terms ($f(M_1, \ldots, M_n)$). We let u range over both names and variables.

3.2   Destructors

Destructors are partial functions that processes can apply to terms, and are ranged over by g. The semantics of destructors is specified by the reduction relation $\Downarrow$: given the terms $M_1, \ldots, M_n$ as arguments, the destructor g can either succeed and provide a term N as a result (which we denote as $g(M_1, \ldots, M_n) \Downarrow N$) or it can fail (denoted as $g(M_1, \ldots, M_n) \not\Downarrow$). The dec destructor decrypts an encrypted message given the corresponding decryption key (dec$($enc$(M, pk(K)), K) \Downarrow M$). The check destructor checks a signed message using a verification key, and if this succeeds returns the message without the signature (check$($sign$(M, K), vk(K)) \Downarrow M$). The application of eq succeeds, yielding the constant true, if the two arguments are syntactically the same (eq$(M, M) \Downarrow$ true). The destructors $\wedge$ and $\vee$ model conjunctions and disjunctions, respectively ($\wedge($true, true$) \Downarrow$ true, $\vee(M, $true$) \Downarrow$ true, and $\vee($true$, M) \Downarrow$ true).

3.3   Representing Zero-knowledge Proofs

Constructing Zero-knowledge Proofs. In a very similar way to what is proposed in [11], a non-interactive zero-knowledge proof of a statement S is represented as a term of the form zk$_{n,m,S}(N_1, \ldots, N_n; M_1, \ldots, M_m)$, where $N_1, \ldots, N_n$ and $M_1, \ldots, M_m$ are two sequences of terms. The proof keeps the terms in $N_1, \ldots, N_n$ secret, while the terms $M_1, \ldots, M_m$ are revealed. For clarity we will use semicolons to separate the secret terms from the public ones, and we will often write $\widetilde{N}$ instead of $N_1, \ldots, N_n$ if n is clear from the context.

Statements. In order to express a wide class of zero-knowledge proofs, comprising for instance proofs of signature verifications and decryptions, we need to use destructors inside logical formulas. Since destructors cannot occur inside terms, we need to define a larger class of objects, called *statements*, that also contains destructors. The set of statements (ranged over by S) is the free algebra built from names, variables, the placeholders α_i and β_j, as well as tuples, constructors and destructors (except for the public$_m$ and ver$_{n,m,l,S}$ destructors introduced below) applied to

other statements. It is easy to see that all terms are also statements. For clarity we distinguish an actual destructor g from its counterpart used within statements by writing the latter as $g^\sharp$. The statement S used in a term $\mathsf{zk}_{n,m,S}(N_1, \ldots, N_n; M_1, \ldots, M_m)$ is called an (n,m)-*statement*. It does not contain names or variables, and uses the placeholders α_i and β_j, with $i \in [1,n]$ and $j \in [1,m]$, to refer to the secret terms N_i and public terms M_j. For instance, the zero-knowledge term

$$\mathsf{zk}_{1,2,\mathsf{eq}^\sharp(\beta_1,\mathsf{dec}^\sharp(\mathsf{enc}(\beta_1,\beta_2),\alpha_1))}(\ k\ ;\ m, \mathsf{pk}(k)\ )$$

proves the knowledge of the decryption key k corresponding to the public encryption key $\mathsf{pk}(k)$. More precisely, the statement reads: "There exists a secret key k such that the decryption of the ciphertext $\mathsf{enc}(m, \mathsf{pk}(k))$ with this key yields m". As mentioned before, m and $\mathsf{pk}(k)$ are revealed by the proof while k is kept secret.

Verifying Zero-knowledge Proofs. The destructor $\mathsf{ver}_{n,m,l,S}$ verifies the validity of a zero-knowledge proof. It takes as arguments a proof together with l terms that are matched against the first l arguments in the public component of the proof. If the proof is valid, then $\mathsf{ver}_{n,m,l,S}$ returns the other $m - l$ public arguments. A proof is valid if and only if the statement obtained by substituting all α_i's and β_j's in S with the corresponding values N_i and M_j evaluates to true. This is formalized as follows:

$$\mathsf{ver}_{n,m,l,S}(\mathsf{zk}_{n,m,S}(\widetilde{N}; M_1, \ldots, M_l, \ldots, M_m), M_1, \ldots, M_l) \Downarrow \langle M_{l+1}, \ldots, M_m \rangle, \text{iff } S\{\widetilde{N}/\widetilde{\alpha}\}\{\widetilde{M}/\widetilde{\beta}\} \Downarrow_\sharp \mathsf{true}$$

The evaluation relation $S \Downarrow_\sharp M$ is defined in terms of the reduction rules for the other destructors[1]. For example $g^\sharp(S_1, \ldots, S_n) \Downarrow_\sharp M$ if $\forall i \in [1,n]$. $S_i \Downarrow_\sharp M_i$ and $g(M_1, \ldots, M_n) \Downarrow M$. The $\Downarrow_\sharp$ relation can in fact be seen as the extension of the $\Downarrow$ relation to statements. For instance, in the protocol from Section 2 we have

$$\mathsf{ver}_{1,2,1,\beta_1=\mathsf{hash}(\alpha_1)}(\mathsf{zk}_{1,2,\beta_1=\mathsf{hash}(\alpha_1)}(m_s; \mathsf{hash}(m_s), m_p), pseudo) \Downarrow \langle m_p \rangle$$

since $(\beta_1 = \mathsf{hash}(\alpha_1))\{m_s/\alpha_1\}\{\mathsf{hash}(m_s)/\beta_1\} \equiv (\mathsf{hash}(m_s) = \mathsf{hash}(m_s)) \Downarrow_\sharp \mathsf{true}$. Notice that in [11] the operational semantics of zero-knowledge proof verification is defined by an infinite equational theory, which needs to be compiled into a convergent rewriting system in order to be suitable to ProVerif. Since we use a type-system to analyze security protocols, in this paper we can take a simpler approach and formalize the verification of zero-knowledge proofs by means of a destructor like ver, whose semantics is not directly expressible in ProVerif.

The destructor public_m yields the public component of a zero-knowledge proof ($\mathsf{public}_m(\mathsf{zk}_{n,m,S}(\widetilde{N}, \widetilde{M})) \Downarrow \langle \widetilde{M} \rangle$). Note that the private component is not revealed by any destructor, which intuitively guarantees the zero-knowledge property of the proofs.

3.4 Processes

Processes are essentially the same as in [27]. The process $\mathsf{out}(M, N).P$ outputs message N on channel M and then behaves as P; the process $\mathsf{in}(M, x).P$ receives a message N from channel M and then behaves as $P\{N/x\}$; the process $!\mathsf{in}(M, x).P$ behaves as an unbounded number of copies of $\mathsf{in}(M, x).P$ executed in parallel; $\mathsf{new}\ a : T.P$ generates a fresh name a of type T and then behaves as P; $P \mid Q$ behaves as P executed in parallel with Q; $\mathbf{0}$ is a process that does nothing; $\mathsf{let}\ x = g(\widetilde{M})\ \mathsf{then}\ P\ \mathsf{else}\ Q$ applies the destructor g to the terms $\widetilde{M}$ and if this succeeds and produces the term N ($g(\widetilde{M}) \Downarrow N$) then the process behaves as $P\{N/x\}$, otherwise ($g(\widetilde{M}) \not\Downarrow$) it behaves as Q; the process $\mathsf{let}\ \langle x_1, \ldots, x_n \rangle = M\ \mathsf{in}\ P$ splits the tuple M into its components.

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[1] This is not circular since the ver destructor cannot appear inside statements.

The processes assume C and assert C, where C is a logical formula, are used to express authorization policies, and do not have any computational significance. Assumptions are used to mark security-related events in processes, such as the intention to authenticate message m_p by the party knowing the secret value m_s (assume Authenticate(m_p, m_s)), and also to express global policies such as:

$$\text{assume } \forall m_p, k, m_s.(\text{Authenticate}(m_p, m_s) \ \wedge \ \text{Trust}(\text{hash}(m_s), k)) \ \Rightarrow \text{Associate}(m_p, k)$$

The scope of assumptions is global, i.e., once an assumption becomes active it affects all processes that run in parallel.

Assertions specify logical formulas that are supposed to be entailed at run-time by the currently active assumptions. For instance, in the protocol from Section 2 the verifier asserts that it can associate trust level k to the message m_p (assert Associate(m_p, k)). In principle it might be possible to implement such assertions as (distributed!) dynamic checks. As in [27], we take a totally different approach here. Our type system guarantees statically that in well-typed protocols all asserted formulas are valid at runtime, even in the presence of an arbitrary adversary.

3.5 Authorization Logic

Our calculus and type system are largely independent of the exact choice of authorization logic. The logic is required to fulfill some standard properties, such as monotonicity, closure under substitution and allowing the replacement of equals by equals. Additionally, statements are not only used by zero-knowledge terms, but they also have a close connection with the formulas in our authorization logic. For this reason, we require that all statements are also formulas in the logic, and we assume that $\text{eq}^\sharp$ corresponds to equality in the authorization logic (simply denoted by "$=$"), while $\wedge^\sharp$ and $\vee^\sharp$ correspond to conjunction and disjunction in the logic, respectively. Furthermore, we add axioms in the logic that correspond to the semantics of destructors (e.g., for decryption we add the formula $\forall m, k. \text{dec}^\sharp(\text{enc}(m, \text{pk}(k)), k) = m$ as an axiom). This ensures that if a statement S evaluates to a term M ($S \Downarrow_\sharp M$) then $S = M$ holds in the logic ($\models S = M$). Under this assumption, from the semantics of the ver destructor we can immediately infer that if $\text{ver}_{n,m,l,S}(\text{zk}_{n,m,S}(\widetilde{N}, M_1, \ldots, M_l, \ldots, M_m), M_1, \ldots, M_l) \Downarrow \langle M_{l+1}, \ldots, M_m \rangle$ then $\models S\{\widetilde{N}/\widetilde{\alpha}\}\{\widetilde{M}/\widetilde{\beta}\}$, which captures the soundness of our construction for zero-knowledge. The complete list of requirements the logic needs to fulfill is given in Proposition A.1 in Appendix A.2.

In our implementation we consider first-order logic with equality as the authorization logic and we use the automated theorem prover SPASS [38] to discharge the proof obligations generated by our type system.

3.6 Notations and Conventions

In the processes $\text{in}(M, x).P$ and $\text{let } x = g(\widetilde{M}) \text{ then } P \text{ else } Q$, the variable x is bound in P. In the process $\text{let } \langle \widetilde{x} \rangle = M \text{ in } P$ the variables $\widetilde{x}$ are bound in P. In $\text{new } a : T.P$ the name a is bound in P. Throughout the paper, we identify any phrase ϕ of syntax up to consistent renaming of bound names and variables. We let $fn(\phi)$ denote the set of free names in ϕ, $fv(\phi)$ the set of free variables, and $free(\phi)$ the set of free names and variables. We say that ϕ is closed if it does not have any free variables. We write $\phi\{\phi'/x\}$ for the outcome of the capture-avoiding substitution of ϕ' for each free occurrence of x in ϕ.

A context is a process with a hole where other processes can be plugged in. An evaluation context $\mathcal{E}$ is a context of the form $\mathcal{E} = \text{new } \widetilde{a} : \widetilde{T}.([\,] \mid P)$ for some process P. We use $\text{new } \widetilde{a} : \widetilde{T}$ to denote a sequence $\text{new } a_1 : T_1 \ldots \text{new } a_k : T_k$ of typed name restrictions and, for the sake of readability, we sometimes use $let\ x := M\ in\ P$ to denote $P\{M/x\}$.

3.7 Modeling the Protocol for Anonymous Trust

With this setup in place we can formally model the protocol for anonymous trust from Section 2 as follows:

$P := \mathsf{new}\ m_p : \mathsf{Un}.(\mathsf{assume}\ \mathsf{Authenticate}(m_p, m_s)\ |\ \mathsf{out}(c, \mathsf{zk}_{1,2,\beta_1=\mathsf{hash}(\alpha_1)}(m_s; \mathsf{hash}(m_s), m_p)))$

$V := \mathsf{assume}\ \mathsf{TrustLevel}(pseudo, k)\ |\ \mathsf{in}(c, w).\ \mathsf{let}\ x = \mathsf{ver}_{1,2,1,\beta_1=\mathsf{hash}(\alpha_1)}(w, pseudo)\ \mathsf{then}$
$\qquad \mathsf{let}\ \langle y_p \rangle = x\ \mathsf{then}\ \mathsf{assert}\ \mathsf{Associate}(y_p, k)$

$PseudoTrust := \mathsf{new}\ m_s : \mathsf{Private}.\ P\ |\ let\ pseudo := \mathsf{hash}(m_s)\ in\ V\ |\ \mathsf{assume}\ Policy$

3.8 Operational Semantics and Safety

As for the pi-calculus, the operational semantics of our calculus is defined in terms of *structural equivalence* ($\equiv$) and *internal reduction* ($\rightarrow$). Structural equivalence captures rearrangements of parallel compositions and restrictions. Internal reduction defines the semantics of communication and destructor application. For the formal definitions of these relations, we refer to Appendix A.3.

A process is safe if and only if all its assertions are entailed by the active assumptions in every protocol execution.

Definition 3.1 (Safety) *A closed process P is* safe *if and only if for every C and Q such that $P \rightarrow^* \mathsf{new}\ \widetilde{a} : \widetilde{T}.(\mathsf{assert}\ C\ |\ Q)$, there exists an evaluation context $\mathcal{E} = \mathsf{new}\ \widetilde{b} : \widetilde{U}.[\ ]\ |\ Q'$ such that $Q \equiv \mathcal{E}[\mathsf{assume}\ C_1\ |\ \ldots\ |\ \mathsf{assume}\ C_n]$, $fn(C) \cap \widetilde{b} = \emptyset$, and we have that $\{C_1, \ldots, C_n\} \models C$.*

A process is robustly safe if it is safe when run in parallel with an arbitrary opponent. As we will see, our type system guarantees that if a process is well-typed, then it is also robustly safe.

Definition 3.2 (Opponent) *A closed process is an* opponent *if it does not contain any* assert *and if the only type occurring therein is* Un.

Definition 3.3 (Robust Safety) *A closed process P is* robustly safe *if and only if $P\ |\ O$ is safe for every opponent O.*

4 Type System

The type system presented in this paper extends a recent type system for statically enforcing authorization policies in distributed systems [27]. This basic type system is described in Sections 4.1 and 4.2. The most important novelty of our type system is the ability to reason about zero-knowledge proofs and Section 4.3 presents this in detail. Section 4.4 explains how our type system works on a fragment of the protocol for anonymous trust. Finally,Section 4.5 summarizes the security guarantees offered by the type system.

4.1 Basic Types

Our type system has the following types: Private is the type of messages that are not revealed to the adversary; Un is the type of messages possibly known to the adversary; $\mathsf{Ch}(T)$ is the type of channels carrying messages of type T. As in [27, 13], tuples are given refinement types of the form $\langle x_1 : T_1, \ldots, x_n : T_n \rangle\{C\}$. The formula C can depend on the variables $x_1, \ldots, x_n$. For example, in the protocol of Section 2, the verification of the zero-knowledge proof yields the term $\langle m_p \rangle$ of type $\langle y : \mathsf{Un} \rangle\{\exists x.\ pseudo = \mathsf{hash}(x)\ \wedge\ \mathsf{Authenticate}(y, x)\}$.

In addition to these base types, we also consider types for the different cryptographic primitives. For digital signatures, $\mathsf{SigKey}(T)$ and $\mathsf{VerKey}(T)$ denote the types of the signing and verification

keys for messages of type T, while $\mathsf{Signed}(T)$ is the type of signed messages of type T. We remark that a key of type $\mathsf{SigKey}(T)$ can only be used to sign messages of type T, where the type T is in general annotated by the user. Similarly, $\mathsf{PubKey}(T)$ and $\mathsf{PrivKey}(T)$ denote the types of the public encryption keys and of the private decryption keys for messages of type T, while $\mathsf{PubEnc}(T)$ is the type of a public-key encryption of a message of type T. The type $\mathsf{Hash}(T)$ denotes the type of a hashed message of type T. In all these cases the type T is usually a refinement type conveying a logical formula. For instance, $\mathsf{SigKey}(\langle x : \mathsf{Private}\rangle\{\mathsf{Ok}(x)\})$ is the type of keys that can be used to sign private messages M for which we statically know that $\mathsf{Ok}(M)$ holds.

4.2   Typing Judgments

We describe the type system in prose here, while all the typing rules are given in Appendix B. The type system relies on four typing judgments: well-formed environment ($\Gamma \vdash \diamond$), subtyping ($\Gamma \vdash T <: U$), term typing ($\Gamma \vdash M : T$), and process typing ($\Gamma \vdash P$), which are described in the following.

Well-formed Environment.   The type system relies on a *typing environment*, which is a list containing name and variable bindings of the form $u : T$, together with formulas of the authorization logic. We denote the formulas in a typing environment Γ by *forms*(Γ). Intuitively, these formulas constitute a safe approximation of the formulas assumed at run-time.

A typing environment is *well-formed*, written $\Gamma \vdash \diamond$, if no name or variable is bound more than once, and if all free names and variables inside the types and formulas appearing in the environment are bound beforehand. All the other typing judgments check that the environment they use is well-formed.

Subtyping. All messages sent to and received from an untrusted channel have type Un, since such channels are considered under the complete control of the adversary. However, a system in which only names and variables of type Un could be communicated over the untrusted network would be too restrictive to be useful. We therefore consider a subtyping relation on types, which allows a term of a subtype to be used in all contexts that require a term of a supertype. This preorder is used to compare types with the special type Un. In particular, we allow messages having a type T that is a subtype of Un, denoted $T <: \mathsf{Un}$, to be sent over the untrusted network, and we say that the type T is *public* in this case. Similarly, we allow messages of type Un that are received from the untrusted network to be used as messages of type U, provided that $\mathsf{Un} <: U$, and in this case we say that type U is *tainted*.

For example, in our type system the types $\mathsf{PubKey}(T)$ and $\mathsf{VerKey}(T)$ are always public, meaning that public-key encryption keys as well as signature verification keys can always be sent over an untrusted channel without compromising the security of the protocol. On the other hand, $\mathsf{PrivKey}(T)$ is public only if T is also public, since sending to the adversary a private key that decrypts confidential messages will most likely compromise the security of the protocol. Finally, type $\mathsf{Private}$ from Section 2 is neither public nor tainted.

Typing Terms. The judgment $\Gamma \vdash M : T$ checks that message M has type T. The type of variables and names is simply looked up in the typing environment. A tuple $\langle M_1, \ldots, M_n\rangle$ has the refinement type $\langle x_1 : T_1, \ldots, x_n : T_n\rangle\{C\}$ if each M_i has type T_i and if additionally the formulas in the typing environment entail $C\{\widetilde{M}/\widetilde{x}\}$. The other cases are as one would expect (see Table 16 in Appendix **??**). For instance, if the message M has type T and the key K has type $\mathsf{SigKey}(T)$ then the signature $\mathsf{sign}(M, K)$ has type $\mathsf{Signed}(T)$.

Typing Processes. The typing judgment $\Gamma \vdash P$ checks whether the process P is well-typed. As we will show in Section 4.5, this guarantees that P is secure against an arbitrary adversary.

The rules for type-checking processes are listed in Table 17 in Appendix **??**. The output process $\mathsf{out}(M, N).P$ is well-typed, if the term M has a channel type $\mathsf{Ch}(T)$, N is of type T and the process P is well-typed. For instance, this guarantees that the adversary can only receive messages of type Un at run-time, since it is initially given only channels of type $\mathsf{Ch}(\mathsf{Un})$. Similarly, the input process $\mathsf{in}(M, x).P$ is well-typed only if M has type $\mathsf{Ch}(T)$ and P is well-typed assuming x of type T. The process $\mathsf{new}\ n : T.P$ is well-typed if P is well-typed assuming n of type T. When type-checking a parallel composition $P \mid Q$, the top-level assumptions in P can be added to the typing environment in which Q is typed, and the top-level assumptions in Q can be added to the environment in which P is typed. This ensures that assumptions have global scope.

The process $\mathsf{assert}\ C$ is well-typed in a typing environment Γ only if $forms(\Gamma) \models C$. Intuitively, this guarantees the safety property of well-typed processes, since $forms(\Gamma)$ represents a safe approximation of the formulas assumed at run-time.

Type-checking the process $\mathsf{let}\ x = g(M_1, \ldots, M_n)\ \mathsf{then}\ P$ differs significantly from [27]. As usual, we need to check whether the arguments $M_1, \ldots, M_n$ have the types required by the destructor, and obtain a new type T for the result of the destructor application. The continuation process P is, however, type-checked in a typing environment extended not only with the binding $x : T$, but also with the logical formula "$x = g^{\sharp}(M_1, \ldots, M_n)$". This can be used for further reasoning in the logic. For instance, when checking that $\Gamma \vdash \mathsf{let}\ x = \mathsf{check}(M, K)\ \mathsf{then}\ P$ we first need to ensure that M has type $\mathsf{Signed}(T)$ and K has type $\mathsf{VerKey}(T)$ for the same T. Then we can type-check the process P in the environment $\Gamma, x : T, x = \mathsf{check}^{\sharp}(M, K)$. This treatment of destructors is simpler and more elegant than the one in [27], and appears to be similar to the typing rules for splitting and matching from [13].

When type-checking a process that splits a tuple $\mathsf{let}\ \langle x_1, \ldots, x_n \rangle = M\ \mathsf{in}\ P$, we need to ensure that M has a refinement type $\langle y_1 : T_1, \ldots, y_n : T_n \rangle \{C\}$. Then the continuation process P is checked in an environment extended with the bindings $x_1 : T_1, \ldots, x_n : T_n$ and two logical formulas. First, we assume the formula $\langle x_1, \ldots, x_n \rangle = M$, which is helpful if the same tuple is split again in P. More important, we assume that the formula in the refinement type holds, after appropriate variable replacement, i.e., the environment is extended with $C\{\widetilde{x}/\widetilde{y}\}$ when checking P. This is sound since the type system guarantees that when creating the tuple $\langle \widetilde{M} \rangle$ the formula $C\{\widetilde{M}/\widetilde{y}\}$ is entailed.

Example. As an example consider the final part of the verifier process in Section 3.7. The variable x holds a tuple with one component. The process splits this tuple and assigns the component to a variable y_p, and then asserts that y_p satisfies the predicate $\mathsf{Associate}(y_p, k)$. The process $\mathsf{let}\ \langle y_p \rangle = x\ \mathsf{in}\ \mathsf{assert}\ \mathsf{Associate}(y_p, k)$ is type-checked in the environment that contains:

$$\Gamma_1 = \ldots, (\forall m_p, k, m_s.(\mathsf{Authenticate}(m_p, m_s) \wedge \mathsf{TrustLevel}(\mathsf{hash}(m_s), k)) \Rightarrow \mathsf{Associate}(m_p, k)),$$
$$\mathsf{TrustLevel}(pseudo, k), x : \langle w : \mathsf{Un} \rangle \{\exists v.\ pseudo = \mathsf{hash}(v) \wedge \mathsf{Authenticate}(w, v)\}.$$

After the tuple is split, the environment becomes:

$$\Gamma_2 = \Gamma_1, y_p : \mathsf{Un}, \langle y_p \rangle = x, \exists v.\ pseudo = \mathsf{hash}(v) \wedge \mathsf{Authenticate}(y_p, v).$$

In order to type-check the $\mathsf{assert}\ \mathsf{Associate}(y_p, k)$, we need to ensure that $forms(\Gamma_2) \models \mathsf{Associate}(y_p, k)$, which holds indeed in the authorization logic.

4.3 Type-checking Zero-knowledge

The main novelty of our type system is the treatment of zero-knowledge proofs.

The Zero-knowledge Type. We give zero-knowledge proofs of the form $\mathsf{zk}_{n,m,S}(\widetilde{N}; \widetilde{M})$ type $\mathsf{ZKProof}_{n,m,S}(\langle y_1 : T_1, \ldots, y_m : T_m \rangle \{\exists x_1, \ldots, x_n.C\})$. This type contains a tuple type listing the

types of the arguments in the public component. The logical formula associated to this type is of the form $\exists x_1, \ldots, x_n.C$, where the arguments in the private component are existentially quantified. The type system guarantees that $C\{\widetilde{N}/\widetilde{x}\}\{\widetilde{M}/\widetilde{y}\}$ is entailed by the formulas in the typing environment.

Subtyping. The zero-knowledge type $\mathsf{ZKProof}_{n,m,S}(\langle \widetilde{y} : \widetilde{T}\rangle\{C\})$ is public only if the types $\widetilde{T}$ of all the public arguments of the proof are also public. This is necessary since the adversary can extract the public component of $\mathsf{zk}_{n,m,S}(\widetilde{N}; \widetilde{M})$ using the public_m destructor. For the same reason, the zero-knowledge type $\mathsf{ZKProof}_{n,m,S}(\langle \widetilde{y} : \widetilde{T}\rangle\{C\})$ is tainted only if $\widetilde{T}$ are tainted as well. Finally, the type $\mathsf{ZKProof}_{n,m,S}(\langle \widetilde{y} : \widetilde{T}\rangle\{C\})$ is a subtype of $\mathsf{ZKProof}_{n,m,S}(\langle \widetilde{y} : \widetilde{T}'\rangle\{C'\})$ in Γ, if T_i is a subtype of T_i' for all i, and if additionally $forms(\Gamma) \cup \{C\} \models C'$.

Type Annotations. Typing all the other cryptographic primitives we consider relies on the type of some key, which the user has to annotate explicitly. Zero-knowledge proofs, however, do not depend in general on any key. This poses a problem since type-checking the verification of zero-knowledge proofs should propagate logical formulas in the typing environment of the verifier, and it is not clear what formulas to consider. For instance, when type-checking a process let $\langle \widetilde{y}\rangle = \mathsf{ver}_{n,m,0,S}(z)$ then P, we can safely assume that the formula $\exists \widetilde{x}.\ S\{\widetilde{x}/\widetilde{\alpha}\}\{\widetilde{y}/\widetilde{\beta}\}$ holds for the continuation process P. This is in fact guaranteed by the operational semantics of the ver destructor[2] (see Section 3.3). Such a formula, however, does not suffice to type-check most examples we have tried, since it does not mention any logical predicate.

In order to solve this problem, we allow the user to provide type annotations for each statement used in the process. For each (n,m)-statement S, this annotation is modeled by binding a distinguished free variable $s_{n,m,S}$ to a type of the form $\mathsf{Stm}(\langle y_1 : T_1, \ldots, y_m : T_m\rangle\{\exists x_1, \ldots, x_n.C\})$ in the initial typing environment. Additionally, the environment contains an implicit binding $s_{n,m,S}^{un} : \mathsf{Stm}(\mathsf{Un})$ used to type-check proofs of S generated by the adversary.

Typing Zero-knowledge Proofs. With this setup in place, we can formalize the typing rule for zero-knowledge proofs:

$$
\frac{
\begin{array}{c}
\Gamma(s_{n,m,S}) = \mathsf{Stm}(\langle y_1 : T_1, \ldots, y_m : T_m\rangle\{\exists x_1, \ldots, x_n.C\}) \\
\forall i \in [1, n].\ \Gamma \vdash N_i : U_i \qquad \Gamma \vdash \langle M_1, \ldots, M_m\rangle : \langle y_1 : T_1, \ldots, y_m : T_m\rangle\{C\{\widetilde{N}/\widetilde{x}\}\}
\end{array}
}{
\begin{array}{c}
\Gamma \vdash \mathsf{zk}_{n,m,S}(N_1, \ldots, N_n; M_1, \ldots, M_m) : \\
\mathsf{ZKProof}_{n,m,S}(\langle y_1 : T_1, \ldots, y_m : T_m\rangle\{\exists x_1, \ldots, x_n.C\})
\end{array}
}
$$

Note that we require that $\Gamma \vdash \langle M_1, \ldots, M_m\rangle : \langle y_1 : T_1, \ldots, y_m : T_m\rangle\{C\{\widetilde{N}/\widetilde{x}\}\}$, which not only makes sure that the public arguments have the required types, but it also effectively checks that $C\{\widetilde{N}/\widetilde{x}\}\{\widetilde{M}/\widetilde{y}\}$ logically follows from the formulas in the typing environment of the prover. This way we ensure that the honest participants can only generate proofs for statements that are associated to formulas already entailed by the environment.

Typing Zero-Knowledge Verification. Suppose that we are given the process let $x = \mathsf{ver}_{n,m,l,S}(N, M_1, \ldots, M_l)$ then P else Q, a typing environment Γ such that $\Gamma(s_{n,m,S}) = \mathsf{Stm}(T)$, for some $T = \langle y_1 : T_1, \ldots, y_m : T_m\rangle\{\exists x_1, \ldots, x_n.C\}$ annotated by the user, and a type T' such that $\Gamma \vdash N : \mathsf{ZKProof}_{n,m,S}(T')$. Zero-knowledge proofs are typically received from channels controlled by the attacker and in this case $T' = \langle y_1 : \mathsf{Un}, \ldots, y_m : \mathsf{Un}\rangle\{\mathsf{true}\}$, with $\mathsf{ZKProof}_{n,m,S}(T')$ being equivalent to Un by subtyping. In order to type-check this process, we first check that the terms $M_1, \ldots, M_l$ have type $T_1, \ldots, T_l$, since these arguments are matched against the first l terms in the public component of the verified proof. Moreover, we need to check that N can be safely given the stronger type $\mathsf{ZKProof}_{n,m,S}(T)$.

---

[2]Notice that we did not match any of the public arguments of the proof.

The main idea for obtaining stronger guarantees than those given by the semantics of the ver destructor is to use the types of the matched public arguments to derive the type of the other arguments of the proof, even the private ones. For instance, if a matched public argument is a hash of type $\mathsf{Hash}(U)$ and the statement proves the knowledge of the value inside, then we can derive that this value has type U. Similarly, if a matched public argument is a key of type $\mathsf{VerKey}(U)$ and the statement proves the verification of a signature using this key, then the message that is signed has type U. This kind of reasoning can be exploited to infer both type information and logical formulas. Furthermore, if we can statically verify that at least one of the arguments of the proof is neither public nor tainted, then we know that the zero-knowledge proof has been generated by a honest participant (since the adversary knows only terms that are public or tainted). This immediately implies that the proof has the stronger type $\mathsf{ZKProof}_{n,m,S}(T)$, since zero-knowledge proofs constructed by honest participants are type-checked against the type specified by the user. The same principle has been applied in [3] for verifying the secrecy of nonce handshakes.

This intuitive reasoning is formalized by the predicate $\langle\!\langle S \rangle\!\rangle_{\Gamma,n,m,l,T,T'}$ (see Table 19 in Appendix B.1). If this predicate holds, then the zero-knowledge proof N of type $\mathsf{ZKProof}_{n,m,S}(T')$ is guaranteed to have the stronger type $\mathsf{ZKProof}_{n,m,S}(T)$ and we can safely give the last $m - l$ arguments of the proof type $\langle y_{l+1} : T_{l+1}, \ldots, y_m : T_m \rangle \{C\{M_i/y_i\}_{i\in[1,l]}\}$. With this setup in place, the typing rule for the ver destructor is defined as follows:

$$\frac{\begin{array}{c} \Gamma \vdash N : \mathsf{ZKProof}_{n,m,S}(T') \\ \Gamma(s_{n,m,S}) = \mathsf{Stm}(T), \ \text{where} \ T = \langle y_1 : T_1, \ldots, y_m : T_m \rangle \{\exists x_1, \ldots, x_n.\ C\} \\ \forall i \in [1,l].\ \Gamma \vdash M_i : T_i \\ \langle\!\langle S \rangle\!\rangle_{\Gamma,n,m,l,T,T'} \ \text{holds} \qquad \Gamma, x : \langle y_{l+1} : T_{l+1}, \ldots, y_m : T_m \rangle \{\exists \widetilde{x}.\ C\{M_i/y_i\}_{i\in[1,l]}\} \vdash P \qquad \Gamma \vdash Q \end{array}}{\Gamma \vdash \mathsf{let}\ x = \mathsf{ver}_{n,m,l,S}(N, M_1, \ldots, M_l)\ \mathsf{then}\ P\ \mathsf{else}\ Q}$$

4.4 Type-checking the Protocol for Anonymous Trust

We explain how to type-check the prover and the verifier processes from Section 3.7, in the initial typing environment: $\Gamma_0 = s_{1,2,\beta_1=\mathsf{hash}(\alpha_1)} : \mathsf{Stm}(T_{zk}), m_s : \mathsf{Private}, c : \mathsf{Un}, k : \mathsf{Un}$, where $T_{zk} = \langle y_1 : \mathsf{Hash}(\mathsf{Private}), y_2 : \mathsf{Un} \rangle \{\exists x.\ y_1 = \mathsf{hash}(x)\ \wedge\ \mathsf{Authenticate}(y_2, x)\}$.

For the prover, we need to check that the term $\mathsf{zk}_{1,2,\beta_1=\mathsf{hash}(\alpha_1)}(m_s; \mathsf{hash}(m_s), m_p)$ has type $\mathsf{ZKProof}_{1,2,\beta_1=\mathsf{hash}(\alpha_1)}(T_{zk})$, in the extended environment $\Gamma_1 = \Gamma_0, m_p : \mathsf{Un}, \mathsf{Authenticate}(m_p, m_s)$. For this we need to show that $\Gamma_1 \vdash \mathsf{hash}(m_s) : \mathsf{Hash}(\mathsf{Private})$, $\Gamma_1 \vdash m_p : \mathsf{Un}$ and $\mathit{forms}(\Gamma_1) \models \exists x.\ \mathsf{hash}(m_s) = \mathsf{hash}(x)\ \wedge\ \mathsf{Authenticate}(m_p, x)$, which holds by instantiating x to m_s.

The verifier receives the zero-knowledge proof from the untrusted[3] channel c. The zero-knowledge proof is thus bound to a variable w of type Un and, by subtyping, also of type $\mathsf{ZKProof}_{1,2,\beta_1=\mathsf{hash}(\alpha_1)}(\langle y_1 : \mathsf{Un}, y_2 : \mathsf{Un}\rangle\{\mathsf{true}\})$. Note that this type is not strong enough to type-check the continuation process. Type-checking the destructor application $\mathsf{ver}_{1,2,1,\beta_1=\mathsf{hash}(\alpha_1)}(w, \mathit{pseudo})$ can however rely on the fact that $\Gamma \vdash \mathit{pseudo} : \mathsf{Hash}(\mathsf{Private})$, and therefore pseudo is the hash of some message m_s of type $\mathsf{Private}$, which is neither public nor tainted. The statement guarantees that the prover knows m_s and this is enough to ensure that the zero-knowledge proof is generated by a honest participant. Therefore $\langle\!\langle S \rangle\!\rangle_{\Gamma,n,m,l,T,T'}$ holds, and the type system gives the result of the destructor application type $\langle y : \mathsf{Un}\rangle\{\exists v.\ \mathit{pseudo} = \mathsf{hash}(v) \wedge \mathsf{Authenticate}(y, v)\}$. This allows type-checking the continuation process as discussed at the end of Section 4.2.

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[3] We consider all free names to have type Un.

4.5 Security Guarantees

Our type system statically guarantees that in well-typed processes all asserted formulas are valid at runtime (safety), even in the presence of an arbitrary adversary (robust safety). We use $\Gamma \vdash_{\sf Un} P$ to denote $\Gamma, u_1 : {\sf Un}, \ldots, u_n : {\sf Un} \vdash P$, where $\{u_1, \ldots, u_n\} = \mathit{free}(P)$.

Theorem 4.1 (Robust Safety) *Let* $\Gamma = s_{n_1, m_1, S_1} : {\sf Stm}(T_1), \ldots, s_{n_k, m_k, S_k} : {\sf Stm}(T_k)$. *For every closed process* P, *if* $\Gamma \vdash_{\sf Un} P$ *then* P *is robustly safe.*

The proof of this theorem and of all the necessary lemmas are given in Appendix C.

5 Implementation

We implemented an automatic type-checker for the type system presented in this paper. The type-checking phase is guaranteed to terminate and generates proof obligations that are discharged independently, leading to a modular and robust analysis. We use first-order logic with equality as the authorization logic and we employ the automated theorem prover SPASS [38] to discharge the proof obligations. Internally, SPASS uses superposition for equational reasoning [7]. Our tool is written in Objective Caml, comprises approximately 3000 lines of code, and is available at [10].

The implementation uses an algorithmic version of our type system. Devising a variant of the type system that is suitable for an implementation required us to eliminate the subsumption rule and to deal with the facts that the constructors and destructors are in fact polymorphic and that the instantiation of the type variables needs to be made automatically. The latter problem is not trivial since our type system features subtyping. On the other hand, asking the user to annotate every constructor and destructor application would have been unacceptable from a usability perspective.

We tested our tool on the DAA protocol (see Section 6) and several simpler examples including the anonymous trust management protocol given in Section 2. The analysis of DAA terminated in less than three seconds and discharged 30 non-trivial proof obligations, while for the simpler examples the time needed was less than half a second. These promising results show that our static analysis technique has the potential to scale up to industrial-size protocols.

6 Case Study: Direct Anonymous Attestation Protocol

To exemplify the applicability of our type system to real-world protocols, we modeled and analyzed the authenticity properties of the Direct Anonymous Attestation protocol (DAA) [16]. DAA constitutes a cryptographic protocol that enables the remote authentication of a hardware module called the Trusted Platform Module (TPM), while preserving the anonymity of the user owning the module. Such TPMs are now included in many end-user notebooks. More precisely, the goal of the DAA protocol is to enable the TPM to sign arbitrary messages and to send them to an entity called the verifier in such a way that the verifier will only learn that a valid TPM signed that message, but without revealing the TPM's identity. The DAA protocol relies heavily on zero-knowledge proofs to achieve this kind of anonymous authentication.

The DAA protocol is composed of two sub-protocols: the *join protocol* and the *DAA-signing protocol*. The join protocol allows a TPM to obtain a certificate from an entity called the issuer. The DAA-signing protocol enables a TPM to authenticate a message and to prove the verifier to own a valid certificate without revealing the TPM's identity. The protocol ensures that even the issuer cannot link the TPM to its subsequently produced DAA-signatures.
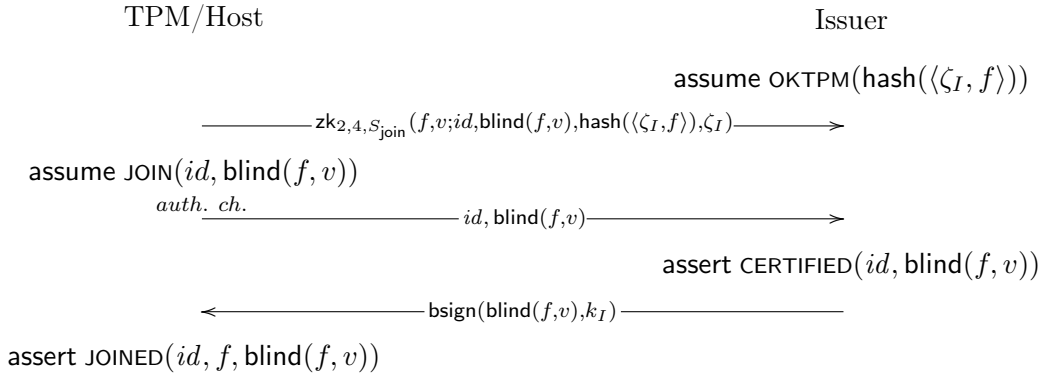
13

Every TPM has a unique *id* as well as a key-pair $(k_{id}, \mathsf{pk}(k_{id}))$ called *endorsement key* (EK). The issuer is assumed to know the public component $\mathsf{pk}(k_{id})$ of each EK. The protocol further assumes the existence a publicly known string bsn_I called the basename of the issuer. Every TPM has a secret seed *daaseed* that allows it to derive secret values $f := \mathsf{hash}_{\mathsf{Private}}\big(\langle \mathsf{hash}_{\mathsf{Private}}(\langle daaseed, \mathsf{hash}(lpk_I)\rangle), cnt, n_0\rangle\big)$, where lpk_I is the long-term public key of the issuer, cnt is a counter, and n_0 is the integer 0. Each such f-value represents a virtual identity with respect to which the TPM can execute the join and the DAA-signing protocol.

In order to prevent the issuer from learning f-values, DAA relies on blind signatures [19]. The idea is that the TPM sends the disguised (or blinded) f-value $\mathsf{blind}(f, r)$, where r is a random blinding factor, to the issuer, which then produces the blind signature $\mathsf{bsign}(\mathsf{blind}(f, r), k_I)$. The TPM can later unblind the signature obtaining a regular signature $\mathsf{sign}(f, k_I)$ of the f-value which can be publicly verified as a regular digital signature. The unblinding of blind signatures is ruled by the $\mathsf{unblind}$ destructor, while the verification of the unblinded signature is denoted by the bcheck destructor. The type $\mathsf{Blind}(T)$ describes blinded messages of type T, $\mathsf{BlindSigKey}(T)$ and $\mathsf{BlindVerKey}(T)$ describe signing and verification keys for blind signatures of messages of type T, $\mathsf{BlindSigned}(T)$ describes blind signatures of messages of type T, and $\mathsf{Blinder}(T)$ describes a blinding factor for messages of type T. DAA additionally relies on secret hashes $\mathsf{hash}_{\mathsf{Private}}(M)$, which are given type $\mathsf{Hash}_{\mathsf{Private}}(T)$. For more details we refer the interested reader to Appendix D.

Table 1 reports the process for the DAA system. For the sake of readability we use $\mathsf{let}\ \langle x_1, \ldots, x_n\rangle = g(\widetilde{M})\ \mathsf{then}\ P\ \mathsf{else}\ Q$ to denote the process $\mathsf{let}\ z = g(\widetilde{M})\ \mathsf{then}\ \mathsf{let}\ \langle x_1, \ldots, x_n\rangle = z\ \mathsf{in}\ P\ \mathsf{else}\ Q$, where z is a fresh variable. We also use $\langle T_1, \ldots, T_n\rangle$ to denote $\langle T_1, \ldots, T_n\rangle\{\mathsf{true}\}$.

6.1 Join protocol

In the join protocol, the TPM can receive a certificate for one of its f-values f from the issuer. The join protocol has the following overall shape:

<center>TPM/Host   Issuer</center>

<center>assume $\mathsf{OKTPM}(\mathsf{hash}(\langle \zeta_I, f\rangle))$</center>

<center>$\xrightarrow{\quad \mathsf{zk}_{2,4,S_{\mathsf{join}}}(f,v;id,\mathsf{blind}(f,v),\mathsf{hash}(\langle \zeta_I,f\rangle),\zeta_I) \quad}$</center>

assume $\mathsf{JOIN}(id, \mathsf{blind}(f, v))$

<center>$\xrightarrow[\quad\quad\quad\quad\quad\quad]{auth.\ ch.\quad\quad id, \mathsf{blind}(f,v)\quad\quad}$</center>

<center>assert $\mathsf{CERTIFIED}(id, \mathsf{blind}(f, v))$</center>

<center>$\xleftarrow{\quad\quad\quad \mathsf{bsign}(\mathsf{blind}(f,v),k_I) \quad\quad\quad}$</center>

assert $\mathsf{JOINED}(id, f, \mathsf{blind}(f, v))$

The TPM sends to the issuer the blinded f-value $\mathsf{blind}(f, v)$, for some random blinding factor v. The TPM is also required to send the hash value $\mathsf{hash}(\langle \zeta_I, f\rangle)$ along with its request where ζ_I is a value derived from the issuer's basename bsn_I. This message is used in a rogue-tagging procedure allowing the issuer to recognize corrupted TPMs. All these messages are transmitted together with a zero-knowledge proof, which guarantees that the f-value f is hashed together with ζ_I in $\mathsf{hash}(\langle \zeta_I, f\rangle)$. The statement of this zero-knowledge proof is modeled as follows:

$$S_{\mathsf{join}} := (\mathsf{blind}(\alpha_1, \alpha_2) = \beta_2\ \wedge\ \mathsf{hash}(\langle \beta_4, \alpha_1\rangle) = \beta_3)$$

The DAA protocol assumes an authentic channel between the TPM and the issuer in order to authenticate the blinded f-value, and the authors suggest a challenge-response handshake based on

Table 1 Our model of DAA

T_f $:= \mathsf{Hash}_{\mathsf{Private}}(\langle x : \mathsf{Hash}_{\mathsf{Private}}(\langle x_1 : \mathsf{Private}, x_2 : \mathsf{Un}\rangle), y : \mathsf{Un}, z : \mathsf{Un}\rangle)$

T_{k_I} $:= \langle y_U : \mathsf{Blind}(T_f)\rangle\{\exists id.\mathsf{CERTIFIED}(id, y_U)\}$

$s_{2,4,S_{\mathsf{join}}}$: $\langle y_{id} : \mathsf{Un}, y_U : \mathsf{Blind}(T_f), y_N : \mathsf{Un}, y_\zeta : \mathsf{Un}\rangle \{\exists x_1, x_2.(y_U = \mathsf{blind}(x_1, x_2) \ \wedge \ y_N = \mathsf{hash}(\langle y_\zeta, x_1\rangle))\}$

$s_{2,4,S_{\mathsf{sign}}}$: $\langle y_{vk} : \mathsf{BlindVerKey}(T_{k_I}), y_N : \mathsf{Un}, y_\zeta : \mathsf{Un}, y_m : \mathsf{Un}\rangle$
         $\{\exists x_f, x_c, x_v, x_{id}. \ y_N = \mathsf{hash}(\langle y_\zeta, x_f\rangle) \wedge \mathsf{CERTIFIED}(x_{id}, \mathsf{blind}(x_f, x_v)) \wedge \ \mathsf{SIGNED}(x_f, y_m)\}$

$\mathsf{P}_{\mathsf{join}}$ $:= \forall id, f, v_1, v_2.(\mathsf{JOIN}(id, \mathsf{blind}(f, v_1)) \wedge \mathsf{OKTPM}(\mathsf{hash}(\langle v_2, f\rangle)) \Rightarrow \mathsf{CERTIFIED}(id, \mathsf{blind}(f, v_1))) \wedge$
                                  $(\exists id'.\mathsf{CERTIFIED}(id', \mathsf{blind}(f, v_1)) \Rightarrow \mathsf{JOINED}(id, f, \mathsf{blind}(f, v_1)))$

$\mathsf{P}_{\mathsf{sign}}$ $:= \forall f, v, m.(\exists id'.\mathsf{CERTIFIED}(id', \mathsf{blind}(f, v)) \wedge \mathsf{SIGNED}(f, m)) \Rightarrow \mathsf{AUTHENTICATED}(m)$

daa $:= \mathit{let}\ S_{\mathsf{join}} := (\mathsf{blind}(\alpha_1, \alpha_2) = \beta_2 \ \wedge \ \mathsf{hash}(\langle \beta_4, \alpha_1\rangle) = \beta_3)\ \mathit{in}$
            $\mathit{let}\ S_{\mathsf{sign}} := (\mathsf{bcheck}(\alpha_2, \beta_1) = \alpha_1 \ \wedge \ \mathsf{hash}(\langle \beta_3, \alpha_1\rangle) = \beta_2)\ \mathit{in}$
            $\mathsf{new}\ k_I : \mathsf{BlindSigKey}(T_{k_I}).\ \mathsf{new}\ daaseed : \mathsf{Private}.\ \mathsf{new}\ k_{id} : \mathsf{PrivKey}(\mathsf{Un}).$
            $\mathit{let}\ f := \mathsf{hash}_{\mathsf{Private}}(\ \langle \mathsf{hash}_{\mathsf{Private}}(\langle daaseed, \mathsf{hash}(lpk_I)\rangle\ ), cnt, n_0\rangle\ )\ \mathit{in}$
            $\mathit{let}\ \zeta_I := \mathsf{hash}(\langle n_1, bsn_I\rangle)\ \mathit{in}$
            $\mathit{let}\ N_I := \mathsf{hash}(\langle \zeta_I, f\rangle)\ \mathit{in}$
            $\mathsf{new}\ authch : \mathsf{Ch}(\langle y_{id} : \mathsf{Un}, y_U : \mathsf{Blind}(T_f)\rangle\{\mathsf{JOIN}(y_{id}, y_U)\}).$
            $(\mathsf{tpm}\ |\ \mathsf{issuer}\ |\ \mathsf{verifier}\ |\ \mathsf{assume}\ \mathsf{P}_{\mathsf{join}}\ |\ \mathsf{assume}\ \mathsf{P}_{\mathsf{sign}})$

tpm :=	issuer :=
$\quad\mathsf{new}\ v : \mathsf{Blinder}(T_f).$	$\quad\mathsf{assume}\ \mathsf{OKTPM}(N_I)\ \|$
$\quad\mathit{let}\ U := \mathsf{blind}(f, v)\ \mathit{in}$	$\quad!\mathsf{in}(pub, zkjoin).$
$\quad(\mathsf{assume}\ \mathsf{JOIN}(id, U)\ \|$	$\quad\mathsf{in}(authch, z).$
$\quad\mathit{let}\ zkjoin := \mathsf{zk}_{2,4,S_{\mathsf{join}}}(f, v; id, U, N_I, \zeta_I)\ \mathit{in}$	$\quad\mathit{let}\ \langle y_{id}, y_U\rangle = z\ \mathit{in}$
$\quad\mathsf{out}(pub, zkjoin).\mathsf{out}(authch, \ \langle id, U\rangle\ ).$	$\quad\mathit{let}\ \langle\rangle = \mathsf{ver}_{2,4,4,S_{\mathsf{join}}}(zkjoin, y_{id}, y_U, N_I, \zeta_I)\ \mathit{then}$
$\quad\mathsf{in}(pub, x).$	$\quad(\mathsf{assert}\ \mathsf{CERTIFIED}(y_{id}, y_U)\ \|$
$\quad\mathit{let}\ cert = \mathsf{unblind}(x, v, \mathsf{bvk}(k_I))\ \mathit{then}$	$\quad\mathsf{out}(pub, \mathsf{bsign}(\langle y_U\rangle, k_I)))$
$\quad\mathit{let}\ \langle x_f\rangle = \mathsf{bcheck}(cert, \mathsf{bvk}(k_I))\ \mathit{then}$	verifier :=
$\quad\mathit{let}\ x' = \mathsf{eq}(x_f, f)\ \mathit{then}$	$\quad!\mathsf{in}(pub, zksign).$
$\quad(\mathsf{assert}\ \mathsf{JOINED}(id, f, U)\ \|$	$\quad\mathit{let}\ \langle x_N, x_\zeta, x_m\rangle = \mathsf{ver}_{2,4,1,S_{\mathsf{sign}}}(zksign, \mathsf{bvk}(k_I))\ \mathit{then}$
$\quad\mathsf{new}\ m : \mathsf{Un}.\ \mathsf{new}\ \zeta : \mathsf{Un}.$	$\quad\mathsf{assert}\ \mathsf{AUTHENTICATED}(x_m)$
$\quad\mathit{let}\ N := \mathsf{hash}(\langle \zeta, f\rangle)\ \mathit{in}$	
$\quad(\mathsf{assume}\ \mathsf{SIGNED}(f, m)\ \|$	
$\quad\mathit{let}\ zksign := \mathsf{zk}_{2,4,S_{\mathsf{sign}}}(f, cert; \mathsf{bvk}(k_I), N, \zeta, m)\ \mathit{in}$	
$\quad\mathsf{out}(pub, zksign))))$	

---

the TPM endorsement key as a possible implementation [16]. Type-checking a challenge-response handshake would require us to introduce challenge-response nonce types similarly to [29, 30, 33]. For the sake of simplicity, we abstract away from the actual cryptographic implementation of such an authentic channel, and we let the TPM send its own identifier together with the blinded f-value over a private channel shared with the issuer. Note that the blinded f-value is still known to the attacker, since it occurs in the public component of the zero-knowledge proof, which is sent over an untrusted channel. Finally, the issuer sends to the TPM the blind signature $\mathsf{bsign}(\mathsf{blind}(f, v), k_I)$.

Type-checking the Join Protocol. The type specified by the user for $s_{2,4,S_{\mathsf{join}}}$ is $\mathsf{Stm}(T_{\mathsf{join}})$, where T_{join} is

$$\langle y_{id} : \mathsf{Un}, y_U : \mathsf{Blind}(T_f), y_N : \mathsf{Un}, y_\zeta : \mathsf{Un}\rangle\{\exists x_1, x_2.(y_U = \mathsf{blind}(x_1, x_2) \ \wedge \ y_N = \mathsf{hash}(\langle y_\zeta, x_1\rangle))\}$$

and the type of the f-value is $T_f := \mathsf{Hash}_{\mathsf{Private}}(\langle x : \mathsf{Hash}_{\mathsf{Private}}(\langle x_1 : \mathsf{Private}, x_2 : \mathsf{Un}\rangle), y : \mathsf{Un}, z : \mathsf{Un}\rangle)$. The formula in T_{join} simply gives a logical characterization of the structure of the messages sent by the TPM to the issuer, which is directly guaranteed by the statement S_{join} of the zero-knowledge proof. Therefore $\langle\!\langle S \rangle\!\rangle_{2,4,4,S_{\mathsf{join}},T_{\mathsf{join}},\mathsf{Un}}$ holds on the verifier's side and the logical formula is inserted into the typing environment. The type of the authentic channel is $\mathsf{Ch}(\langle y_{id} : \mathsf{Un}, y_U : \mathsf{Blind}(T_f)\rangle\{\mathsf{JOIN}(y_{id}, y_U)\})$. The type system guarantees that the TPM assumes $\mathsf{JOIN}(id, U)$ before sending id and the blinded f-value U on such a channel. Finally, the type of the issuer's signing key is

$$\langle y_U : \mathsf{Blind}(T_f)\rangle\{\exists id.\mathsf{CERTIFIED}(id, y_U)\}$$
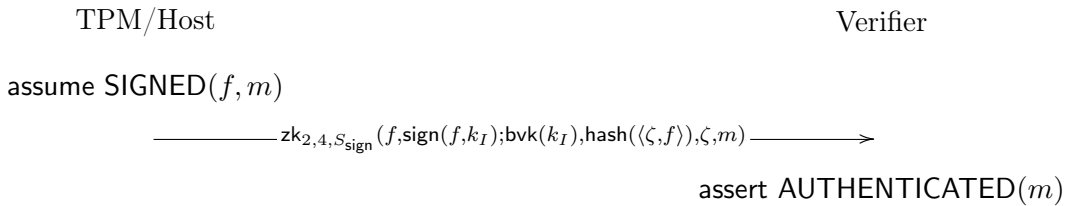
The type system guarantees that whenever the issuer releases a certificate for message M, M is a blinded secret and there exists id such that $\mathsf{CERTIFIED}(id, M)$ is entailed by the formulas in the typing environment. The authorization policy for the join protocol is as follows:

$$\forall id, f, v_1, v_2.(\mathsf{JOIN}(id, \mathsf{blind}(f, v_1)) \wedge \mathsf{OKTPM}(\mathsf{hash}(\langle v_2, f\rangle)) \Rightarrow \mathsf{CERTIFIED}(id, \mathsf{blind}(f, v_1))) \wedge$$
$$(\exists id'.\mathsf{CERTIFIED}(id', \mathsf{blind}(f, v_1)) \Rightarrow \mathsf{JOINED}(id, f, \mathsf{blind}(f, v_1)))$$

This policy allows the issuer to release a blind signature for TPM id (assertion $\mathsf{CERTIFIED}(id, \mathsf{blind}(f, v_1))$) only if the TPM id has started the join protocol to authenticate $\mathsf{blind}(f, v_1)$ (assumption $\mathsf{JOIN}(id, \mathsf{blind}(f, v_1))$) and the f-value f is associated to a valid TPM (assumption $\mathsf{OKTPM}(\mathsf{hash}(\langle v_2, f\rangle))$). Additionally, the policy guarantees that whenever a TPM id successfully completes the join protocol (assertion $\mathsf{JOINED}(id, f, \mathsf{blind}(f, v_1))$), the issuer has certified $\mathsf{blind}(f, v_1)$ (assertion $\exists id'.\mathsf{CERTIFIED}(id', \mathsf{blind}(f, v_1))$).

6.2 DAA-signing protocol

After successfully executing the join protocol, the TPM has a valid certificate for its f-value f signed by the issuer. Since only valid TPMs should be able to DAA-sign a message m, the TPM has to convince a verifier that it possesses a valid certificate. Of course, the TPM cannot directly send it to the verifier, since this would reveal f. Instead, the TPM produces *zksign*, a zero-knowledge proof that it knows a valid certificate. If the TPM would, however, just send (*zksign*, m) to the verifier, the protocol would be subject to a trivial message substitution attack. Message m is instead combined with the proof so that one can only replace m by redoing the proof (and this again can only be done by knowing a valid certificate). The overall shape of the DAA-signing protocol is hence as follows:

TPM/Host	Verifier

assume $\mathsf{SIGNED}(f, m)$

$$\xrightarrow{\quad \mathsf{zk}_{2,4,S_{\mathsf{sign}}}(f, \mathsf{sign}(f, k_I); \mathsf{bvk}(k_I), \mathsf{hash}(\langle \zeta, f\rangle), \zeta, m)\quad}$$

assert $\mathsf{AUTHENTICATED}(m)$

with $S_{\mathsf{sign}} := (\mathsf{bcheck}(\alpha_2, \beta_1) = \alpha_1 \wedge \mathsf{hash}(\langle \beta_3, \alpha_1\rangle) = \beta_2)$. The zero-knowledge proof guarantees that the secret f-value f is signed by the issuer and that such a value is hashed together with a fresh value ζ[4]. This hash is used in the rogue tagging procedure mentioned above.

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[4]In the pseudonymous variant of the DAA-signing protocol ζ is derived in a deterministic fashion from the basename bsn_V of the verifier. Our analysis can be easily adapted to this variant.

Type-checking the DAA-signing Protocol. The type specified by the user for $s_{2,4,S_{\text{sign}}}$ is $\text{Stm}(T_{\text{sign}})$, where

$$T_{\text{sign}} := \begin{array}{l} \langle y_{vk} : \text{BlindVerKey}(T_{k_I}), y_N : \text{Un}, y_\zeta : \text{Un}, y_m : \text{Un} \rangle \\ \{\exists x_f, x_c, x_v, x_{id}.\ y_N = \text{hash}(\langle y_\zeta, x_f \rangle) \wedge \text{CERTIFIED}(x_{id}, \text{blind}(x_f, x_v)) \wedge\ \text{SIGNED}(x_f, y_m)\} \end{array}$$

This type guarantees that the f-value of the TPM has been certified by the issuer (assertion $\text{CERTIFIED}(x_{id}, \text{blind}(x_f, x_v))$, captures the constraint on the hash inherited from the statement of the zero-knowledge proof $(y_N = \text{hash}(\langle y_\zeta, x_f \rangle))$, and states that the user has signed message m (assumption $\text{SIGNED}(x_f, y_m)$). On the verifier's side, the assertion $\text{CERTIFIED}(x_{id}, \text{blind}(x_f, x_v))$ is guaranteed to hold by the verification of the certificate proved by zero-knowledge and by the type of the verification key, while the equality $y_N = \text{hash}(\langle y_\zeta, x_f \rangle)$ is enforced by the semantics of the ver destructor. Furthermore the type of the verification key guarantees that the f-value is of type T_f. Since values of this type are neither public nor tainted, the proof is generated by a honest TPM, and thus $\langle\!\langle S \rangle\!\rangle_{2,4,4,S_{\text{sign}},T_{\text{sign}},\text{Un}}$ holds, and the logical formula is inserted into the typing environment. The authorization policy for the DAA-signing protocol is:

$$\forall f, v, m.(\exists id'.\text{CERTIFIED}(id', \text{blind}(f, v)) \wedge \text{SIGNED}(f, m)) \Rightarrow \text{AUTHENTICATED}(m)$$

This policy allows the verifier to authenticate message m (assertion $\text{AUTHENTICATED}(m)$) only if the sender proves the knowledge of some certified f-value f associated to some TPM id (assertion $\exists id'.\text{CERTIFIED}(id', \text{blind}(f, v)))$ and the zero-knowledge proof includes message m (assumption $\text{SIGNED}(f, m)$). Note that the id of the TPM is existentially quantified, since it is not known to the verifier.

Our type-checker can prove in less than three seconds that $s_{2,4,S_{\text{join}}} : \text{Stm}(T_{\text{join}}), s_{2,4,S_{\text{sign}}} : \text{Stm}(T_{\text{sign}}) \vdash_{\text{Un}} \text{daa}$. By Theorem 4.1, this guarantees that process daa is robustly safe.

7 Security Despite Compromise

In this section we discuss about the security of partially compromised systems, in which some of the participants are under the control of the adversary. We show that zero-knowledge proofs can be used to prove correct behavior to remote parties, which can safely derive authorization decisions based on these proofs.

The DAA protocol guarantees the intended authenticity property even if the TPM is corrupted. Intuitively, this property is guaranteed by the zero-knowledge proof used in the DAA-signing protocol: the TPM can generate this proof only if the f-value has been previously certified by the issuer. One might wonder whether the authorization policy for the DAA-signing protocol holds if we type-check the TPM code as an opponent, i.e., by removing all assumptions and assertions and giving all messages, including *daaseed* and the f-value f, type Un.

The authorization policy and the type of the zero-knowledge proof for the DAA-signing protocol are given in Section 6.2. The first part of the policy $(\exists id'.\text{CERTIFIED}(id', blind(f, v)))$ is guaranteed by the zero-knowledge proof itself. No matter whether the TPM is corrupted or not, the proof guarantees that the TPM knows a signature of type $\text{Signed}(\langle x : T_f \rangle \{\exists y, id.\text{CERTIFIED}(id, \text{blind}(x, y))\})$. The type of the verification key alone allows the verifier to check the type of the signature (cf. the predicate for the verification of statements extended to blind signatures in Appendix D), even if this signature is in the private component of the zero-knowledge proof.

The second part of authorization policy (i.e., $\text{SIGNED}(f, m)$) holds only if the TPM is not corrupted. If the TPM is corrupted, then the adversary can generate a valid zero-knowledge proof without assuming $\text{SIGNED}(f, m)$. As expected, this variant of DAA would not type-check

because the statement verification would no longer allow the verifier to derive $\mathsf{SIGNED}(f, m)$, since f, the content of the signature, would have type Un instead of T_f. This scenario should, however, not be regarded as an attack since the adversary is just following the protocol. This suggests that, in order to prove that DAA is secure against TPM compromise, we could actually weaken the authorization policy as follows:

$$\forall f, v, m.(\exists id'.\mathsf{CERTIFIED}(id', blind(f, v)) \;\wedge\; \mathsf{VERIFIED}(f, cert; \mathsf{bvk}(k_I), N, \zeta, m))$$
$$\Rightarrow \mathsf{AUTHENTICATED}(m)$$

Whenever the verification of a zero-knowledge proof of the form $\mathsf{zk}_{n,m,S}(M_1, \ldots, M_n; N_1, \ldots, N_m)$ succeeds we introduce a new assumption $\exists x_1, \ldots, x_n.\mathsf{VERIFIED}(x_1, \ldots, x_n, N_1, \ldots, N_m)$ that is automatically added to the process and justified in the typing environment. This can be achieved by changing the operational semantics of the ver destructor and refining the corresponding typing rule. The special assumption $\mathsf{VERIFIED}$ binds the private component and the public component of a successfully verified zero-knowledge proof. In the example above, $\mathsf{VERIFIED}(f, \ldots, m)$ binds the secret f-value f and the message m in the zero-knowledge proof of the DAA-signing protocol. After this modification, DAA can be successfully verified, even if the TPM is type-checked as an opponent. The same holds for the simple protocol introduced in Section 2, if the prover is type-checked as an opponent.

In general, we notice that some formulas are guaranteed to hold by the verification of a zero-knowledge proof whether the prover is corrupted or not. These formulas are the ones obtained from the validity of cryptographic operations proved by zero-knowledge (e.g., the assumption $\exists id'.\mathsf{CERTIFIED}(id', blind(f, v))$ obtained from the verification of the issuer' signature in DAA), and the special formula $\exists x_1, \ldots, x_n.\mathsf{VERIFIED}(x_1, \ldots, x_n, N_1, \ldots, N_n)$. We argue that this may be used to systematically enforce authenticity properties on partially compromised systems.

8   Conclusions

This paper shows how certain security properties of zero-knowledge proofs can be characterized as authorization policies and statically enforced by a novel type system. The zero-knowledge proofs are given dependent types where the messages kept secret by the proof are existentially quantified in the logic. We developed a type-checker for this type system and use an automated theorem-prover to discharge proof obligations. We applied our technique to verify the authenticity properties of the Direct Anonymous Attestation (DAA) protocol.

The analysis technique is very efficient. Furthermore, the combination of types and authorization logics constitutes a truly expressive framework to model and analyze a variety of trace-based security properties. Zero-knowledge proofs perfectly fit into this framework, offering the possibility to implement fine-grained authorization policies that rely on the existential quantification in the logic. This is particularly well-suited for protocols for privacy and anonymity.

The type system for authorization proposed by Fournet et al. [27], which our approach is grounded on, has been recently applied to the analysis of protocol implementations written in F# [13]. We are confident that our technique could be incorporated into such a framework in order to verify implementations of protocols based on zero-knowledge proofs.

In Section 7 we explore the usage of zero-knowledge proofs in the design of systems that guarantee security despite partial compromise. We believe that zero-knowledge proofs are the natural candidate for strengthening protocol specifications against compromised participants, since they can be used to verify the correct behavior of remote parties and to safely derive authorization decisions. A formal elaboration of these ideas is an interesting direction for future research.

Finally, while there is no restriction on the shape of the statements that can be proved in our abstract setting, finding a sound and efficient cryptographic implementation for these symbolic zero-knowledge proofs is far from trivial. In [12] the authors identify the properties a concrete zero-knowledge proof system should satisfy in order to soundly implement a simpler abstraction of zero-knowledge, and mention two existing cryptographic constructions that satisfy these properties. The efficiency and expressivity of these constructions have, however, not been thoroughly studied and constitute interesting topics for further investigation.

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A   Applied Pi-calculus with Destructors

A.1   Terms, Statements, Destructors and Processes

In this paper we consider a variant of the applied pi-calculus with constructors and destructors, similar to the ones in [4, 27].

Thesyntax of *terms* is defined in Table 2, together with the list of *constructors* we consider. *Statements* are defined in Table 3. The destructors we consider are given in Table 4 and their semantics is formalized by the relation $g(M_1, \ldots, M_n) \Downarrow N$. The semantics of the ver destructor additionally relies on the statement evaluation relation given in Table 5. Finally, the syntax of *processes* is reported in Table 6. Notice that we restrict the replications to appear only in front of inputs, as done in [27], since this simplifies the proofs.

Table 2 Terms and constructors

$K, L, M, N ::=$		terms
	a, b, c, m, n, k	names
	x, y, z, v, w	variables
	$\langle M_1, \ldots, M_n \rangle$	tuple
	$f(M_1, \ldots, M_n)$	constructor application (f of arity n)

$$f \quad ::= \quad \mathsf{pk}^1, \mathsf{enc}^2, \mathsf{vk}^1, \mathsf{sign}^2, \mathsf{hash}^1, \mathsf{true}^0, \mathsf{zk}_{n,m,S}^{n+m}$$

Note: $\mathsf{zk}_{n,m,S}$ is only defined when S is an (n, m)-statement.

Table 3 Syntax of statements

$S, P ::=$		statements
	a, b, m, n, k	names
	x, y, z, v, w	variables
	α_i, β_i	placeholders
	$\langle S_1, \ldots, S_n \rangle$	tuple
	$f(S_1, \ldots, S_n)$	constructor application (f of arity n)
	$g^\sharp(S_1, \ldots, S_n)$	destructor application (g of arity n)

Note: $g^\sharp$ is only defined if $g \neq \mathsf{ver}$.

Table 4 Destructors $\hfill g(M_1, \ldots, M_n) \Downarrow N$

$$g \quad ::= \quad \mathsf{eq}^2, \wedge^2, \vee^2, \mathsf{dec}^2, \mathsf{check}^2, \mathsf{public}_m^1, \mathsf{ver}_{n,m,l,S}^{l+1}.$$

Note: $\mathsf{ver}_{n,m,l,S}^{l+1}$ is only defined when S is an (n, m)-statement and $l \in [1, m]$.

$$
\begin{array}{llll}
\mathsf{dec}(\mathsf{enc}(M, \mathsf{pk}(K)), K) & \Downarrow & M & \qquad \mathsf{eq}(M, M) \quad \Downarrow \quad \mathsf{true} \\
\mathsf{check}(\mathsf{sign}(M, K), \mathsf{vk}(K)) & \Downarrow & M & \qquad \wedge(\mathsf{true}, \mathsf{true}) \quad \Downarrow \quad \mathsf{true} \\
\mathsf{public}_m(\mathsf{zk}_{n,m,S}(\widetilde{N}, \widetilde{M})) & \Downarrow & \langle \widetilde{M} \rangle & \qquad \vee(M, \mathsf{true}) \quad \Downarrow \quad \mathsf{true} \\
& & & \qquad \vee(\mathsf{true}, M) \quad \Downarrow \quad \mathsf{true}
\end{array}
$$

$$\mathsf{ver}_{n,m,l,S}(\mathsf{zk}_{n,m,S}(\widetilde{N}, M_1, \ldots, M_l, \ldots, M_m), M_1, \ldots, M_l) \Downarrow \langle M_{l+1}, \ldots, M_m \rangle \text{ iff } S\{\widetilde{N}/\widetilde{\alpha}\}\{\widetilde{M}/\widetilde{\beta}\} \Downarrow_\sharp \mathsf{true}$$

Notation: $\widetilde{M} = M_1, \ldots, M_n$.
Notation: We write $g(M_1, \ldots, M_n) \not\Downarrow$ if none of the rules above applies, i.e., the destructor fails.

Table 5 Evaluation of statements $\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad S \Downarrow_\sharp M$

Ev-Term
$$M \Downarrow_\sharp M$$

Ev-Constr
$$\frac{\forall i \in [1,n].\ S_i \Downarrow_\sharp M_i}{f(S_1,\ldots,S_n) \Downarrow_\sharp f(M_1,\ldots,M_n)}$$

Ev-Tuple
$$\frac{\forall i \in [1,n].\ S_i \Downarrow_\sharp M_i}{\langle S_1,\ldots,S_n\rangle \Downarrow_\sharp \langle M_1,\ldots,M_n\rangle}$$

Ev-Destr
$$\frac{\forall i \in [1,n].\ S_i \Downarrow_\sharp M_i \qquad g(M_1,\ldots,M_n) \Downarrow M}{g^\sharp(S_1,\ldots,S_n) \Downarrow_\sharp M}$$

Note: Please note that $S \Downarrow_\sharp M$ is only defined when S does not contain α's and β's.

Table 6 Syntax of processes

$P, Q, R \quad ::=$		processes
	$\mathsf{out}(M, N).P$	output
	$\mathsf{in}(M, x).P$	input
	$!\mathsf{in}(M, x).P$	replicated input
	$\mathsf{new}\ a : T.P$	restriction
	$P \mid Q$	parallel composition
	$\mathbf{0}$	null process
	$\mathsf{let}\ x = g(\widetilde{M})\ \mathsf{then}\ P\ \mathsf{else}\ Q$	destructor evaluation
	$\mathsf{let}\ \langle x_1,\ldots,x_n\rangle = M\ \mathsf{in}\ P$	pair splitting
	$\mathsf{assume}\ C$	assume formula
	$\mathsf{assert}\ C$	expect formula to hold

A.2 Authorization Logic

We assume that the logical entailment relation $A \models C$ is expansive, monotonous, idempotent and closed under substitution of terms for variables. The equality in the logic needs to be an equivalence relation, and the logic must allow replacing equals by equals. With the exception of the rule for ver, the destructor reduction rules need to be valid equalities in the logic (e.g., they can be added as axioms). The logic needs to support function symbols and tuples; however, they do not need to be first-class, it is enough if we can encode them faithfully (e.g., it is easy to encode tuples in first-order logic). Finally, the logic is assumed to include some of the usual logical connectives with their canonical meaning: true, false, $\wedge$, $\vee$, and $\exists$. We do not assume, however, the logic to be classical – an intuitionistic logic could also fulfill the assumptions below.

Proposition A.1 (Logical Entailment: Assumptions)

Expansivity: *$C \in A$ implies that $A \models C$;*

Monotonicity: *$A \models C$ and $A \subseteq A'$ then $A' \models C$;*

Idempotence: *$A \models A'$ and $A \cup A' \models C$ then $A \models C$;*

Substitution: *$A \models C$ then $A\sigma \models C\sigma$;*

Reflexivity: *$\models M = M$;*

Symmetry: *If $A \models N = M$ then $A \models M = N$;*

Transitivity: *If $A \models N = M$ and $A \models M = L$ then $A \models N = L$;*

Replacement: *$A \models M = N$ and $A \models C\{M/x\}$ imply that also $A \models C\{N/x\}$;*

Destructors: *If $g(M_1, \ldots, M_n) \Downarrow N$ and $g \neq$ ver then $\models g^\sharp(M_1, \ldots, M_n) = N$.*

Tuples: *If $A \models \langle M_1, \ldots, M_n \rangle = \langle N_1, \ldots, N_n \rangle$ then $\forall i \in [1, n].\ A \models M_i = N_i$;*

True: *$\models$ true;*

False: *false $\models A$;*

Equals-True: *$A \models C =$ true if and only if $A \models C$;*

And: *$A \models C \wedge C'$ if and only if $A \models C$ and $A \models C'$;*

Or: *$A \models C \vee C'$ if and only if $A \models C$ or $A \models C'$;*

Existential: *If $A \models C\{M/x\}$ then $A \models \exists x.C$*

A.3   Operational Semantics

The semantics of the calculus is standard and is defined by the usual structural equivalence $(P \equiv Q)$ and an internal reduction relation $(P \rightarrow Q)$.

Structural equivalence relates the processes that are considered equivalent up to syntactic re-arrangement. It is the smallest equivalence relation satisfying the rules in Table 7.

Table 7 Structural equivalence $\hspace{6cm} P \equiv Q$

$$
\begin{array}{lrcl}
(\textsc{Eq-Zero-Id}) & P \mid \mathbf{0} & \equiv & P \\
(\textsc{Eq-Par-Comm}) & P \mid Q & \equiv & Q \mid P \\
(\textsc{Eq-Par-Assoc}) & (P \mid Q) \mid R & \equiv & P \mid (Q \mid R) \\
(\textsc{Eq-Scope}) & \mathsf{new}\ a : T.(P \mid Q) & \equiv & P \mid \mathsf{new}\ a : T.Q,\ \textit{if } a \notin \textit{fn}(P) \\
(\textsc{Eq-Bind-Swap}) & \mathsf{new}\ a_1 : T_1.\mathsf{new}\ a_2 : T_2.P & \equiv & \mathsf{new}\ a_2 : T_2.\mathsf{new}\ a_1 : T_1.P,\ \textit{if } a_1 \neq a_2 \\
(\textsc{Eq-Ctxt}) & \mathcal{E}[P] & \equiv & \mathcal{E}[Q],\ \textit{if } P \equiv Q \\
\end{array}
$$

Where $\mathcal{E}$ stands for an evaluation context, i.e., a context of the form $\mathcal{E} = \mathsf{new}\ \widetilde{a} : \widetilde{T}.([\ ] \mid P)$.

Internal reduction defines the semantics of process synchronizations and conditionals. It is the smallest relation on closed processes satisfying the rules in Table 8.

Table 8 Internal reduction $\hspace{6cm} P \rightarrow Q$

$$
\begin{array}{lrcl}
(\textsc{Red-I/O}) & \mathsf{out}(a, M).P \mid \mathsf{in}(a, x).Q & \rightarrow & P \mid Q\{M/x\} \\
(\textsc{Red-!I/O}) & \mathsf{out}(a, M).P \mid\ !\mathsf{in}(a, x).Q & \rightarrow & P \mid Q\{M/x\} \mid\ !\mathsf{in}(a, x).Q \\
(\textsc{Red-Destr}) & \mathsf{let}\ x = g(\widetilde{M})\ \mathsf{then}\ P\ \mathsf{else}\ Q & \rightarrow & P\{N/x\},\ \ \textit{if } g(\widetilde{M}) \Downarrow N \\
(\textsc{Red-Else}) & \mathsf{let}\ x = g(\widetilde{M})\ \mathsf{then}\ P\ \mathsf{else}\ Q & \rightarrow & Q,\ \ \textit{if } g(\widetilde{M}) \not\Downarrow \\
(\textsc{Red-Split}) & \mathsf{let}\ \langle \widetilde{x} \rangle = \langle \widetilde{M} \rangle\ \mathsf{in}\ P & \rightarrow & P\{\widetilde{M}/\widetilde{x}\} \\
(\textsc{Red-Ctxt}) & \mathcal{E}[P] & \rightarrow & \mathcal{E}[Q],\ \ \textit{if } P \rightarrow Q \\
(\textsc{Red-Eq}) & P \rightarrow Q, & & \textit{if } P \equiv P', P' \rightarrow Q',\ \text{and } Q' \equiv Q \\
\end{array}
$$

B The Type System for Zero-knowledge

Table 9 lists the types of our type system, while in Table 10 we list the typing judgements. The well-formed environment judgement is defined in Table 11.

The kinding rules are given in Table 12, and the subtyping rules in Table 13. Compared to [29, 27] we make a distinction between the types for public-key encryption and the ones for digital signatures, which allows for slightly more expressive subtyping for these primitives.

Tables 14 and 15 are devoted to the types of some of the constructors and destructors. In Table 16 we define the term typing judgement. Table 17 lists the rules for typing processes, which are defined using the auxiliary statement verification predicate (Table 19) and the environment extraction relation (Table 18).

Table 9 Types

$$
\begin{array}{llllll}
T, U ::= & \mathsf{Un} & \mathsf{Ch}(T) & \mathsf{SigKey}(T) & \mathsf{PubKey}(T) & \mathsf{Hash}(T) \\
& \mathsf{Private} & \langle \widetilde{x} : \widetilde{T} \rangle \{C\} & \mathsf{VerKey}(T) & \mathsf{PrivKey}(T) & \mathsf{Stm}(T) \\
& \top & & \mathsf{Signed}(T) & \mathsf{PubEnc}(T) & \mathsf{ZKProof}_{n,m,S}(T)
\end{array}
$$

Notation: Let $\widetilde{x} : \widetilde{T}$ denote $x_1 : T_1, \ldots, x_n : T_n$ for some n.
Note: For the refinement type $\langle \widetilde{x} : \widetilde{T} \rangle \{C\}$ the variables $\widetilde{x}$ are bound in C, but not bound in $\widetilde{T}$.

Table 10 Typing Judgements

$$
\begin{array}{ll}
\Gamma \vdash \diamond & \text{well-formed environment} \\
\Gamma \vdash T :: k & \text{kinding, } k \in \{\mathsf{pub}, \mathsf{tnt}\} \\
\Gamma \vdash T <: U & \text{subtyping} \\
f : (T_1, \ldots, T_n) \mapsto T & \text{constructor typing} \\
g : (T_1, \ldots, T_n) \mapsto T & \text{destructor typing} \\
\Gamma \vdash M : T & \text{term typing} \\
\Gamma \vdash P & \text{well-typed process}
\end{array}
$$

Notation: We use $\Gamma \vdash \mathcal{J}$ to denote a typing judgement where $\mathcal{J} \in \{\diamond, T :: k, T <: U, M : T, P\}$

Table 11 Well-formed environment $\qquad\qquad\qquad \Gamma \vdash \diamond$

ENV-EMPTY
$$\emptyset \vdash \diamond$$

ENV-FORMULA
$$\frac{\Gamma \vdash \diamond \qquad \mathit{free}(C) \subseteq \mathit{dom}(\Gamma)}{\Gamma, C \vdash \diamond}$$

ENV-BINDING
$$\frac{\Gamma \vdash \diamond \qquad u \notin \mathit{dom}(\Gamma) \qquad \mathit{free}(T) \subseteq \mathit{dom}(\Gamma)}{\Gamma, u : T \vdash \diamond}$$

Definition: $\mathit{dom}(\emptyset) = \emptyset$; $\mathit{dom}(\Gamma, C) = \mathit{dom}(\Gamma)$; $\mathit{dom}(\Gamma, u : T) = \mathit{dom}(\Gamma), u$

Convention: We will very often disregard the order in $\mathit{dom}(\Gamma)$ and use it as a set.

Definition: $\mathit{binds}(\emptyset) = \emptyset$; $\mathit{binds}(\Gamma, C) = \mathit{binds}(\Gamma)$; $\mathit{binds}(\Gamma, u : T) = \mathit{dom}(\Gamma), u : T$

Definition: $\mathit{forms}(\emptyset) = \emptyset$; $\mathit{forms}(\Gamma, C) = \mathit{forms}(\Gamma) \cup \{C\}$; $\mathit{forms}(\Gamma, u : T) = \mathit{forms}(\Gamma)$

Table 12 Kinding ($k \in \{\mathsf{pub}, \mathsf{tnt}\}$) $\hfill \Gamma \vdash T :: k$

KIND-UN
$$\frac{\Gamma \vdash \diamond}{\Gamma \vdash \mathsf{Un} :: k}$$

KIND-CHAN
$$\frac{\Gamma \vdash T :: \mathsf{pub} \qquad \Gamma \vdash T :: \mathsf{tnt}}{\Gamma \vdash \mathsf{Ch}(T) :: k}$$

KIND-TUPLE-PUB
$$\frac{\forall i.\ \Gamma \vdash T_i :: \mathsf{pub} \qquad \Gamma, \widetilde{x} : \widetilde{T}, C \vdash \diamond}{\Gamma \vdash \langle \widetilde{x} : \widetilde{T} \rangle \{C\} :: \mathsf{pub}}$$

KIND-TUPLE-TNT
$$\frac{\forall i.\ \Gamma \vdash T_i :: \mathsf{tnt} \qquad \Gamma, \widetilde{x} : \widetilde{T}, C \vdash \diamond \qquad \mathit{forms}(\Gamma, \widetilde{x} : \widetilde{T}) \models C}{\Gamma \vdash \langle \widetilde{x} : \widetilde{T} \rangle \{C\} :: \mathsf{tnt}}$$

KIND-SIGNED-PUB
$$\frac{\Gamma \vdash T :: \mathsf{pub}}{\Gamma \vdash \mathsf{Signed}(T) :: \mathsf{pub}}$$

KIND-SIGNED-TNT
$$\frac{\Gamma \vdash \diamond \qquad \mathit{free}(T) \subseteq \mathit{dom}(\Gamma)}{\Gamma \vdash \mathsf{Signed}(T) :: \mathsf{tnt}}$$

KIND-SIGKEY
$$\frac{\Gamma \vdash T :: \mathsf{tnt}}{\Gamma \vdash \mathsf{SigKey}(T) :: k}$$

KIND-VERKEY-PUB
$$\frac{\Gamma \vdash \diamond \qquad \mathit{free}(T) \subseteq \mathit{dom}(\Gamma)}{\Gamma \vdash \mathsf{VerKey}(T) :: \mathsf{pub}}$$

KIND-VERKEY-TNT
$$\frac{\Gamma \vdash T :: \mathsf{tnt}}{\Gamma \vdash \mathsf{VerKey}(T) :: \mathsf{tnt}}$$

KIND-PUBENC-PUB
$$\frac{\Gamma \vdash \diamond \qquad \mathit{free}(T) \subseteq \mathit{dom}(\Gamma)}{\Gamma \vdash \mathsf{PubEnc}(T) :: \mathsf{pub}}$$

KIND-PUBENC-TNT
$$\frac{\Gamma \vdash T :: \mathsf{tnt}}{\Gamma \vdash \mathsf{PubEnc}(T) :: \mathsf{tnt}}$$

KIND-PUBKEY-PUB
$$\frac{\Gamma \vdash \diamond \qquad \mathit{free}(T) \subseteq \mathit{dom}(\Gamma)}{\Gamma \vdash \mathsf{PubKey}(T) :: \mathsf{pub}}$$

KIND-PUBKEY-TNT
$$\frac{\Gamma \vdash T :: \mathsf{pub}}{\Gamma \vdash \mathsf{PubKey}(T) :: \mathsf{tnt}}$$

KIND-PRIVKEY
$$\frac{\Gamma \vdash T :: \mathsf{pub}}{\Gamma \vdash \mathsf{PrivKey}(T) :: k}$$

KIND-HASH-PUB
$$\frac{\Gamma \vdash \diamond \qquad \mathit{free}(T) \subseteq \mathit{dom}(\Gamma)}{\Gamma \vdash \mathsf{Hash}(T) :: \mathsf{pub}}$$

KIND-HASH-TNT
$$\frac{\Gamma \vdash T :: \mathsf{tnt}}{\Gamma \vdash \mathsf{Hash}(T) :: \mathsf{tnt}}$$

KIND-ZK
$$\frac{\forall i \in [1, m].\ \Gamma \vdash T_i :: k \qquad \Gamma, \widetilde{y} : \widetilde{T}, C \vdash \diamond}{\Gamma \vdash \mathsf{ZKProof}_{n,m,S}(\langle y_1 : T_1, \ldots, y_m : T_m \rangle \{C\}) :: k}$$

Table 13 Subtyping $\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\quad \Gamma \vdash T <: U$

Sub-Pub-Tnt
$$\frac{\Gamma \vdash T :: \mathsf{pub} \qquad \Gamma \vdash U :: \mathsf{tnt}}{\Gamma \vdash T <: U}$$

Sub-Refl
$$\frac{\Gamma \vdash \diamond \qquad free(T) \subseteq dom(\Gamma)}{\Gamma \vdash T <: T}$$

Sub-Top
$$\frac{\Gamma \vdash \diamond \qquad free(T) \subseteq dom(\Gamma)}{\Gamma \vdash T <: \top}$$

Sub-Tuple
$$\frac{\forall i.\ \Gamma \vdash T_i <: U_i \qquad \Gamma, \widetilde{x} : \widetilde{T}, C \vdash \diamond \qquad forms(\Gamma, \widetilde{x} : \widetilde{T}) \cup \{C\} \models C'}{\Gamma \vdash \langle \widetilde{x} : \widetilde{T}\rangle\{C\} <: \langle \widetilde{x} : \widetilde{U}\rangle\{C'\}}$$

Sub-Chan-Inv
$$\frac{\Gamma \vdash T <:> U}{\Gamma \vdash \mathsf{Ch}(T) <: \mathsf{Ch}(U)}$$

Sub-Signed-Inv
$$\frac{\Gamma \vdash T <:> U}{\Gamma \vdash \mathsf{Signed}(T) <: \mathsf{Signed}(U)}$$

Sub-SigKey-Inv
$$\frac{\Gamma \vdash T <:> U}{\Gamma \vdash \mathsf{SigKey}(T) <: \mathsf{SigKey}(U)}$$

Sub-VerKey-Cov
$$\frac{\Gamma \vdash T <: U}{\Gamma \vdash \mathsf{VerKey}(T) <: \mathsf{VerKey}(U)}$$

Sub-PubEnc-Inv
$$\frac{\Gamma \vdash T <:> U}{\Gamma \vdash \mathsf{PubEnc}(T) <: \mathsf{PubEnc}(U)}$$

Sub-PubKey-Con
$$\frac{\Gamma \vdash U <: T}{\Gamma \vdash \mathsf{PubKey}(T) <: \mathsf{PubKey}(U)}$$

Sub-PrivKey-Inv
$$\frac{\Gamma \vdash T <:> U}{\Gamma \vdash \mathsf{PrivKey}(T) <: \mathsf{PrivKey}(U)}$$

Sub-Hash-Inv
$$\frac{\Gamma \vdash T <:> U}{\Gamma \vdash \mathsf{Hash}(T) <: \mathsf{Hash}(U)}$$

Sub-Zk-Cov
$$\frac{\Gamma \vdash T <: U}{\Gamma \vdash \mathsf{ZKProof}_{n,m,S}(T) <: \mathsf{ZKProof}_{n,m,P}(U)}$$

Notation: $\Gamma \vdash T <:> U$ iff $\Gamma \vdash T <: U$ and $\Gamma \vdash U <: T$.

Table 14 Typing Constructors $\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\quad f : (T_1, \ldots, T_n) \mapsto U$

$$\mathsf{pk} : (\mathsf{PrivKey}(T)) \mapsto \mathsf{PubKey}(T)$$
$$\mathsf{enc} : (T, \mathsf{PubKey}(T)) \mapsto \mathsf{PubEnc}(T)$$
$$\mathsf{vk} : (\mathsf{SigKey}(T)) \mapsto \mathsf{VerKey}(T)$$
$$\mathsf{sign} : (T, \mathsf{SigKey}(T)) \mapsto \mathsf{Signed}(T)$$
$$\mathsf{hash} : (T) \mapsto \mathsf{Hash}(T)$$
$$\mathsf{true} : () \mapsto \mathsf{Un}$$
$$\mathsf{false} : () \mapsto \mathsf{Un}$$

Table 15 Typing Destructors $\hspace{6cm} g : (T_1, \ldots, T_n) \mapsto U$

$$\mathsf{eq} : (T, T) \mapsto \mathsf{Un}$$
$$\wedge : (\mathsf{Un}, \mathsf{Un}) \mapsto \mathsf{Un}$$
$$\vee : (\mathsf{Un}, \mathsf{Un}) \mapsto \mathsf{Un}$$
$$\mathsf{dec} : (\mathsf{PubEnc}(T), \mathsf{PrivKey}(T)) \mapsto T$$
$$\mathsf{check} : (\mathsf{Signed}(T), \mathsf{VerKey}(T)) \mapsto T$$
$$\mathsf{public}_{n,m} : (\mathsf{ZKProof}_{n,m,S}(\langle y_1 : T_1, \ldots, y_m : T_m\rangle\{C\})) \mapsto \langle y_1 : T_1, \ldots, y_m : T_m\rangle\{\mathsf{true}\}$$

Table 16 Typing Terms $\hspace{8cm} \Gamma \vdash M : T$

Env
$$\frac{\Gamma \vdash \diamond \qquad u : T \in \Gamma}{\Gamma \vdash u : T}$$

Sub
$$\frac{\Gamma \vdash M : T \qquad \Gamma \vdash T <: T'}{\Gamma \vdash M : T'}$$

Constr
$$\frac{f : (T_1, \ldots, T_n) \mapsto T \qquad f \neq \mathsf{zk} \qquad \forall i \in [1, n].\ \Gamma \vdash M_i : T_i}{\Gamma \vdash f(M_1, \ldots, M_n) : T}$$

Tuple
$$\frac{\forall i \in [1, n].\ \Gamma \vdash M_i : T_i \qquad \Gamma, C\{\widetilde{M}/\widetilde{x}\} \vdash \diamond \qquad forms(\Gamma) \models C\{\widetilde{M}/\widetilde{x}\}}{\Gamma \vdash \langle M_1, \ldots, M_n\rangle : \langle x_1 : T_1, \ldots, x_n : T_n\rangle\{C\}}$$

ZK
$$\Gamma(s_{n,m,S}) = \mathsf{Stm}(\langle y_1 : T_1, \ldots, y_m : T_m\rangle\{\exists x_1, \ldots, x_n . C\})$$
$$\frac{\forall i \in [1, n].\ \Gamma \vdash N_i : U_i \qquad \Gamma \vdash \langle M_1, \ldots, M_m\rangle : \langle y_1 : T_1, \ldots, y_m : T_m\rangle\{C\{\widetilde{N}/\widetilde{x}\}\}}{\begin{array}{c}\Gamma \vdash \mathsf{zk}_{n,m,S}(N_1, \ldots, N_n; M_1, \ldots, M_m) : \\ \mathsf{ZKProof}_{n,m,S}(\langle y_1 : T_1, \ldots, y_m : T_m\rangle\{\exists x_1, \ldots, x_n . C\})\end{array}}$$

ZK-Un
$$\frac{\Gamma(s_{n,m,S}^{un}) = \mathsf{Stm}(\mathsf{Un}) \qquad \forall i \in [1, n].\ \Gamma \vdash N_i : \mathsf{Un} \qquad \forall j \in [1, m].\ \Gamma \vdash M_i : \mathsf{Un}}{\Gamma \vdash \mathsf{zk}_{n,m,S}(N_1, \ldots, N_n; M_1, \ldots, M_m) : \mathsf{ZKProof}_{n,m,S}(\langle y_1 : \mathsf{Un}, \ldots, y_m : \mathsf{Un}\rangle\{\exists x_1, \ldots, x_n . \mathsf{true}\})}$$

Notation: We write $\Gamma(u)$ for the type T such that $\Gamma = \Gamma_1, u : T, \Gamma_2$ for some Γ_1 and Γ_2.

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Table 17 Typing Processes $\Gamma \vdash P$

PROC-OUT
$$\frac{\Gamma \vdash M : \mathsf{Ch}(T) \qquad \Gamma \vdash N : T \qquad \Gamma \vdash P}{\Gamma \vdash \mathsf{out}(M, N).P}$$

PROC-(REPL)-IN
$$\frac{\Gamma \vdash M : \mathsf{Ch}(T) \qquad \Gamma, x : T \vdash P}{\Gamma \vdash [!]\mathsf{in}(M, x).P}$$

PROC-STOP
$$\frac{\Gamma \vdash \diamond}{\Gamma \vdash \mathbf{0}}$$

PROC-NEW
$$\frac{T \in \{\mathsf{Un}, \mathsf{Ch}(U), \mathsf{SigKey}(U), \mathsf{PrivKey}(U), \mathsf{Private}\} \qquad \Gamma, a : T \vdash P}{\Gamma \vdash \mathsf{new}\ a : T.P}$$

PROC-PAR
$$\frac{P \rightsquigarrow \Gamma_P \qquad \Gamma, \Gamma_P \vdash Q \qquad Q \rightsquigarrow \Gamma_Q \qquad \Gamma, \Gamma_Q \vdash P}{\Gamma \vdash P \mid Q}$$

PROC-DES
$$\frac{g : (T_1, \ldots, T_n) \mapsto T \qquad g \neq \mathsf{ver} \qquad \forall i \in [1, n].\ \Gamma \vdash M_i : T_i \qquad \Gamma, x : T, x = g^\sharp(M_1, \ldots, M_n) \vdash P \qquad \Gamma \vdash Q}{\Gamma \vdash \mathsf{let}\ x = g(M_1, \ldots, M_n)\ \mathsf{then}\ P\ \mathsf{else}\ Q}$$

PROC-VER
$$\frac{\begin{array}{c} \Gamma \vdash N : \mathsf{ZKProof}_{n,m,S}(T') \\ \Gamma(s_{n,m,S}) = \mathsf{Stm}(T),\ \text{where}\ T = \langle y_1 : T_1, \ldots, y_m : T_m \rangle\{\exists x_1, \ldots, x_n.\ C\} \\ \forall i \in [1, l].\ \Gamma \vdash M_i : T_i \\ \langle\!\langle S \rangle\!\rangle_{\Gamma,n,m,l,T,T'}\ \text{holds} \qquad \Gamma, x : \langle y_{l+1} : T_{l+1}, \ldots, y_m : T_m \rangle\{\exists \widetilde{x}.\ C\{M_i/y_i\}_{i \in [1,l]}\} \vdash P \qquad \Gamma \vdash Q \end{array}}{\Gamma \vdash \mathsf{let}\ x = \mathsf{ver}_{n,m,l,S}(N, M_1, \ldots, M_l)\ \mathsf{then}\ P\ \mathsf{else}\ Q}$$

PROC-VER-UN
$$\frac{\Gamma(s_{n,m,S}^{un}) = \mathsf{Stm}(\mathsf{Un}) \qquad \Gamma \vdash N : \mathsf{Un} \qquad \forall i \in [1, l].\ \Gamma \vdash M_i : \mathsf{Un} \qquad \Gamma, x : \mathsf{Un} \vdash P \qquad \Gamma \vdash Q}{\Gamma \vdash \mathsf{let}\ x = \mathsf{ver}_{n,m,l,S}(N, M_1, \ldots, M_l)\ \mathsf{then}\ P\ \mathsf{else}\ Q}$$

PROC-SPLIT
$$\frac{\Gamma \vdash M : \langle y_1 : T_1, \ldots, y_n : T_n \rangle\{C\} \qquad \Gamma, x_1 : T_1, \ldots, x_n : T_n, \langle x_1, \ldots, x_n \rangle = M, C\{\widetilde{x}/\widetilde{y}\} \vdash P}{\Gamma \vdash \mathsf{let}\ \langle x_1, \ldots, x_n \rangle = M\ \mathsf{in}\ P}$$

PROC-ASSUME
$$\frac{\Gamma, C \vdash \diamond}{\Gamma \vdash \mathsf{assume}\ C}$$

PROC-ASSERT
$$\frac{\Gamma \vdash \diamond \qquad \mathit{forms}(\Gamma) \models C}{\Gamma \vdash \mathsf{assert}\ C}$$

Table 18 Environment Extraction $P \rightsquigarrow \Gamma$

EXTR-NEW
$$\frac{P \rightsquigarrow \Gamma_P}{\mathsf{new}\ a : T.P \rightsquigarrow a : T, \Gamma_P}$$

EXTR-PAR
$$\frac{P \rightsquigarrow \Gamma_P \qquad Q \rightsquigarrow \Gamma_Q}{P \mid Q \rightsquigarrow \Gamma_P, \Gamma_Q}$$

EXTR-ASSUME
$$\mathsf{assume}\ C \rightsquigarrow C$$

EXTR-EMPTY
$$P \rightsquigarrow \emptyset$$

B.1 Statement Verification

As discussed in Section 4.3, the typing rule for the verification of a zero-knowledge proof of the form $\mathsf{zk}_{n,m,S}(N_1, \ldots, N_n; M_1, \ldots, M_m)$ relies on the predicate $\langle\!\langle S \rangle\!\rangle_{\Gamma,n,m,l,T,U}$ in case the verification succeeds, where Γ is the current typing environment of the verifier, l is the number of public messages pattern-matched in the verification destructor, $\mathsf{Stm}(T)$ is the type specified by the user for the (n,m)-statement S, and $\mathsf{ZKProof}_{n,m,S}(U)$ is the type assigned to the zero-knowledge proof before the verification. The predicate $\langle\!\langle S \rangle\!\rangle_{\Gamma,n,m,l,T,U}$ holds only if the zero-knowledge proof is actually of the stronger type $\mathsf{ZKProof}_{n,m,S}(T)$ (we have that $\mathsf{ZKProof}_{n,m,S}(T) <: \mathsf{ZKProof}_{n,m,S}(U)$). Without loss of generality we assume that $T = \langle y_1 : T_1, \ldots, y_m : T_m\rangle\{\exists x_1, \ldots, x_n.C\}$ and $U = \langle y_1 : U_1, \ldots, y_m : U_m\rangle\{\mathsf{true}\}$. The formalization is reported in Table 19.

The predicate $\langle\!\langle S \rangle\!\rangle_{\Gamma,n,m,l,T,U}$ relies on the auxiliary function $[|S\{\widetilde{x}/\widetilde{\alpha}\}\{\widetilde{y}/\widetilde{\beta}\}|]_{\Gamma'}^{y_{l+1}:T_{l+1},\ldots,y_m:T_m;C}$ (which we will often abbreviate as $[|S|]_{\Gamma'}$), where Γ' extends Γ with type bindings for the fresh variables $x_1, \ldots, x_n$ and $y_1, \ldots, y_m$, corresponding to the arguments in the private and public component of the zero-knowledge proof, respectively. The variables $x_1, \ldots, x_n$ are given type $\top$ (top) in Γ', since no type information is available for them. The variables $y_1, \ldots, y_l$ are given types $T_1, \ldots, T_l$, since the first l messages in the public component are pattern-matched in the verification destructor with values having types $T_1, \ldots, T_l$. Finally, the variables $y_{l+1}, \ldots, y_m$ are given types $U_{l+1}, \ldots, U_m$, as specified in the weaker type $\mathsf{ZKProof}_{n,m,S}(U)$ assigned to the zero-knowledge proof before verification. The function $[|S|]_{\Gamma'}$ is described in detail in Section B.2. Intuitively, it returns a typing environment Γ'' that refines the type bindings in Γ' so that if $\Gamma'' \vdash x_i : T_i'$, then $\Gamma \vdash N_i : T_i'$, and similarly if $\Gamma'' \vdash y_j : T_j$, then $\Gamma \vdash M_j : T_j$. The function exploits the assumption that verification succeeds, so by the operational semantics $S\{\widetilde{N}/\widetilde{\alpha}\}\{\widetilde{M}/\widetilde{\beta}\}$ holds true. This is used to partially infer the types of the messages in the private and public components of the zero-knowledge proof. Please note that the messages in the private component are not known to the verifier.

With this setup in place, we say that the predicate $\langle\!\langle S \rangle\!\rangle_{\Gamma,n,m,l,T,U}$ holds if $[|S\{\widetilde{x}/\widetilde{\alpha}\}\{\widetilde{y}/\widetilde{\beta}\}|]_{\Gamma'}^{y_{l+1}:T_{l+1},\ldots,y_m:T_m;C}$ returns a typing environment Γ'' such that the formulas in Γ'' entail C and the variables $y_{l+1}, \ldots, y_m$, corresponding to the public messages that are returned by the verification destructor, have types $T_{l+1}, \ldots, T_m$ in Γ''. In other words, Γ'' proves that the zero-knowledge proof has the stronger type $\mathsf{ZKProof}_{n,m,S}(\langle y_1 : T_1, \ldots, y_m : T_m\rangle\{\exists x_1, \ldots, x_n.C\})$ specified by the user.

B.2 Inferring the type of terms in zero-knowledge proofs

The function $[|S|]_{\Gamma'}$ is defined by induction on the structure of the statement S. If the statement is a conjunction of the form $S_1 \wedge^\sharp S_2$, then we know that both S_1 and S_2 hold true and we can thus combine together the typing information and the formulas obtained from each of the statements. Formally, the function yields the typing environment composed of the intersection of the type bindings and the conjunction of the formulas in the typing environments $[|S_1|]_{[|S_2|]_{\Gamma'}}$ and $[|S_2|]_{[|S_1|]_{\Gamma'}}$. The intersection of two type bindings $x : T$ and $x : U$ is defined only if T and U are comparable and it returns $x : T$ if $T <: U$, and $x : U$ otherwise; this definition is extended to typing environments as expected. Note that the information obtained from a first processing of S_2 is used when processing S_1 and vice versa. This symmetrical formulation ensures that the order of the conjuncts in a statement is not relevant.

In case the statement is a disjunction $S_1 \vee^\sharp S_2$, then we do not know which statement between S_1 and S_2 holds true, and we can thus only infer typing information and formulas that are guaranteed by both the statements. Therefore the function yields the typing environment

composed of the union of the type bindings and the disjunction of the formulas in the typing environments $[|S_1|]_{\Gamma'}$ and $[|S_2|]_{\Gamma'}$.

In the following, we let v range over $x_1, \ldots, x_n$ and $y_1, \ldots, y_m$ and let σ denote $\{\widetilde{N}/\widetilde{\alpha}\}\{\widetilde{M}/\widetilde{\beta}\}$, i.e., the substitution that replaces each place-holder in the statement S by the corresponding argument of the proof. Suppose that the argument of the function $[|S|]_{\Gamma'}$ is a statement of the form $\mathsf{check}^\sharp(v_M, v_K) = v_N$. If the key v_K has type $\mathsf{VerKey}(T^*)$ in Γ' and T^* is untainted but public, then the function returns $\Gamma'[v_N : T^*], \mathsf{check}^\sharp(v_M, v_K) = v_N$. Since T^* is not tainted, the signing key is not known to the attacker. Because $\mathsf{check}(v_M\sigma, v_K\sigma) = v_N\sigma$ holds, the type system guarantees that $v_N\sigma$ has type T^* in Γ'. We can thus safely strengthen the type binding for v_N and include the formula $\mathsf{check}^\sharp(v_M, v_K) = v_N$ in the typing environment. Note that we are checking whether type T^* is *unconditionally untainted*, i.e., $\Gamma', \mathsf{false} \nvdash T^* :: \mathsf{tnt}$. This implies that $\Gamma' \nvdash T^* :: \mathsf{tnt}$ but our stronger formulation additionally ensures that the statement verification predicate is closed under environment extension, so that the standard weakening lemma holds.

If the key v_K has type $\mathsf{VerKey}(T^*)$ in Γ' and T^* is neither tainted *nor public*, then the function returns $\Gamma[v_N : T^*, y_{l+1} : T_{l+1}, \ldots, y_m : T_m], C, \mathsf{check}^\sharp(v_M, v_K) = v_N$. Because $\mathsf{check}(v_M\sigma, v_K\sigma) = v_N\sigma$ holds, the type system guarantees that the message $v_N\sigma$ is neither tainted nor public. Intuitively, since $v_N\sigma$ occurs in the zero-knowledge proof (either in the private or in the public component) and the attacker knows only public messages, the prover must be a well-typed participant. This guarantees that the type of the zero-knowledge proof is $\mathsf{ZKProof}_{n,m,S}(T)$ and we can thus safely refine the type binding for $y_{l+1}, \ldots, y_m$ and include the formulas $C, \mathsf{check}^\sharp(v_M, v_K) = v_N$ in the typing environment. Similar reasoning applies to decryptions and hashes. Other cases can be added to this function in order to deal with additional cryptographic primitives.

Table 19 Statement Verification

$\langle\!\langle\, S \,\rangle\!\rangle_{\Gamma,n,m,l,\langle y_1:T_1,\ldots,y_m:T_m\rangle\{\exists x_1,\ldots,x_n.C\},\langle y_1:U_1,\ldots,y_m:U_m\rangle\{\mathsf{true}\}}$ holds

  if $[|S\{\widetilde{x}/\widetilde{\alpha}\}\{\widetilde{y}/\widetilde{\beta}\}|]^{y_{l+1}:T_{l+1},\ldots,y_m:T_m;C}_{\Gamma'} = \Gamma''$, $forms(\Gamma'') \models C$ and $\forall j \in [l+1,m]. \Gamma'' \vdash y_j : T_j$

  where $\Gamma' = \Gamma, \widetilde{x} : \widetilde{\top}, y_1 : T_1, \ldots, y_l : T_l, y_{l+1} : U_{l+1}, \ldots, y_m : U_m$ and $\Gamma' \vdash \diamond$

Let v range over $\widetilde{x} \cup \widetilde{y}$. We write $[|S|]_{\Gamma'}$ to denote $[|S|]^{y_{l+1}:T_{l+1},\ldots,y_m:T_m;C}_{\Gamma'}$

$[|S_1 \wedge^{\sharp} S_2|]_{\Gamma'} = binds([|S_1|]_{[|S_2|]_{\Gamma'}}) \sqcap binds([|S_2|]_{[|S_1|]_{\Gamma'}}), (\bigwedge_{C \in forms([|S_1|]_{[|S_2|]_{\Gamma'}}) \cup forms([|S_2|]_{[|S_1|]_{\Gamma'}})} C)$

$[|S_1 \vee^{\sharp} S_2|]_{\Gamma'} = binds([|S_1|]_{\Gamma'}) \sqcup binds([|S_2|]_{\Gamma'}), ((\bigwedge_{C \in forms([|S_1|]_{\Gamma'})} C) \vee (\bigwedge_{C \in forms([|S_2|]_{\Gamma'})} C))$

$[|\mathsf{check}^{\sharp}(v_M,v_K) = v_N|]_{\Gamma'} = \Gamma'[v_N : T^*], \mathsf{check}^{\sharp}(v_M,v_K) = v_N$

      if $\Gamma'(v_K) = \mathsf{VerKey}(T^*)$ and $\Gamma', \mathsf{false} \nvdash T^* :: \mathsf{tnt}$ and $\Gamma' \vdash T^* :: \mathsf{pub}$

$[|\mathsf{check}^{\sharp}(v_M,v_K) = v_N|]_{\Gamma'} = \Gamma'[v_N : T^*, y_{l+1} : T_{l+1}, \ldots, y_m : T_m], C, \mathsf{check}^{\sharp}(v_M,v_K) = v_N$

      if $\Gamma'(v_K) = \mathsf{VerKey}(T^*)$ and $\Gamma', \mathsf{false} \nvdash T^* :: \mathsf{tnt}$ and $\Gamma', \mathsf{false} \nvdash T^* :: \mathsf{pub}$

$[|\mathsf{dec}^{\sharp}(v_M,v_K) = v_N|]_{\Gamma'} = \Gamma'[v_N : T^*, y_{l+1} : T_{l+1}, \ldots, y_m : T_m], C, \mathsf{dec}^{\sharp}(v_M,v_K) = v_N$

      if $\Gamma'(v_M) = \mathsf{PubEnc}(T^*)$ and $\Gamma', \mathsf{false} \nvdash T^* :: \mathsf{tnt}$ and $\Gamma', \mathsf{false} \nvdash T^* :: \mathsf{pub}$

$[|v_M = \mathsf{hash}(v_N)|]_{\Gamma'} = \Gamma'[v_N : T^*, y_{l+1} : T_{l+1}, \ldots, y_m : T_m], C, v_M = \mathsf{hash}(v_N)$

      if $\Gamma'(v_M) = \mathsf{Hash}(T^*)$ and $\Gamma', \mathsf{false} \nvdash T^* :: \mathsf{tnt}$ and $\Gamma', \mathsf{false} \nvdash T^* :: \mathsf{pub}$

$[|\langle v_{M_1}, \ldots, v_{M_k} \rangle = v_N|]_{\Gamma'} = \Gamma'[v_{M_1} : U_1, \ldots, v_{M_k} : U_k], C^*\{\widetilde{v_M}/\widetilde{z}\}, \langle v_{M_1}, \ldots, v_{M_k} \rangle = v_N$

        if $\Gamma'(v_N) = \langle z_1 : U_1, \ldots, z_k : U_k \rangle\{C^*\}$

$[|S|]_{\Gamma'} = \Gamma', S$ otherwise

Definitions: For all Γ_1 and Γ_2 such that $dom(\Gamma_1) = dom(\Gamma_2)$ (same variables and names in the same order) and $forms(\Gamma_1) = forms(\Gamma_2) = \emptyset$, we define $\Gamma_1 \sqcap \Gamma_2$ and $\Gamma_1 \sqcup \Gamma_2$ as follows:

$$\emptyset \sqcap \emptyset = \emptyset \qquad (\Gamma_1, u:T) \sqcup (\Gamma_2, u:U) = (\Gamma_1 \sqcup \Gamma_2), u:T \sqcup_{(\Gamma_1 \sqcup \Gamma_2)} U$$
$$\emptyset \sqcup \emptyset = \emptyset \qquad (\Gamma_1, u:T) \sqcap (\Gamma_2, u:U) = (\Gamma_1 \sqcap \Gamma_2), u:T \sqcap_{(\Gamma_1 \sqcap \Gamma_2)} U$$

Definitions:

$$T \sqcap_\Gamma U = \begin{cases} T & \text{if } \Gamma \vdash T <: U \\ U & \text{otherwise if } \Gamma \vdash U <: T \end{cases} \qquad T \sqcup_\Gamma U = \begin{cases} U & \text{if } \Gamma \vdash T <: U \\ T & \text{otherwise if } \Gamma \vdash U <: T \end{cases}$$

Definition:

$$\Gamma[v : U] = \begin{cases} \Gamma', v : U, \Gamma'' & \text{if } \Gamma = \Gamma', v : T, \Gamma'' \\ \Gamma, v : U & \text{otherwise} \end{cases}$$

Note: All functions in this table are partial.

C Soundness Proofs

The proof technique is standard [29, 13].

We first show that typing judgements can be only derived for well-formed typing environments.

Lemma C.1 (Well-formed Environment) *If $\Gamma \vdash \mathcal{J}$, then $\Gamma \vdash \diamond$.*

Proof. By induction on the length of the typing derivation of $\Gamma \vdash \mathcal{J}$. All base cases are trivial, since there we explicitly check that the typing environment is well-formed. $\square$

The weakening lemma states that if a typing environment Γ proves a judgement $\mathcal{J}$, then every well-formed extension of Γ still proves $\mathcal{J}$. Thus, extending the set of typing assumptions means extending the set of provable judgements.

Lemma C.2 (Weakening) *If $\Gamma, \Gamma'' \vdash \mathcal{J}$ and $\Gamma, \Gamma', \Gamma'' \vdash \diamond$, then $\Gamma, \Gamma', \Gamma'' \vdash \mathcal{J}$.*

Proof. The only typing rule explicitly checking for the absence of a term in the typing environment is ENV-BINDING and, by hypothesis, $\Gamma, \Gamma', \Gamma'' \vdash \diamond$. For the rules that have $forms(\Gamma) \models C$ as a premise we use the monotonicity of the logical entailment relation. The proof is by induction on the length of the typing derivation of $\Gamma, \Gamma'' \vdash \mathcal{J}$. $\square$

The strengthening lemma states that if a judgement $\mathcal{J}$ can be proved by a typing environment Γ, then the judgement $\mathcal{J}$ can be proved after removing from Γ the bindings of names and variables not occurring in $\mathcal{J}$, and the logical formulas that are already entailed by the others. In the following, we write $\{C_1, \ldots, C_m\} \models \{C_1', \ldots, C_n'\}$ to denote $\{C_1, \ldots, C_m\} \models C_1' \wedge \ldots \wedge C_n'$.

Lemma C.3 (Strengthening) *If $\Gamma, \Gamma'', \Gamma' \vdash \mathcal{J}$, $dom(\Gamma'') \cap free(\mathcal{J}) = \emptyset$, $forms(\Gamma, \Gamma') \models forms(\Gamma'')$ and $\Gamma, \Gamma' \vdash \diamond$, then $\Gamma, \Gamma' \vdash \mathcal{J}$.*

Proof. The proof proceeds by induction on the length of the typing derivation of $\Gamma, \Gamma'', \Gamma' \vdash \mathcal{J}$, and case analysis on the last applied rule. For rule ENV, we know that $\Gamma, \Gamma'', \Gamma' \vdash u : T$ and thus by the premises of the rule that $(u : T) \in \Gamma, \Gamma'', \Gamma'$. Since $dom(\Gamma'') \cap \{u\} = \emptyset$ we have that $(u : T) \in \Gamma, \Gamma'$. From the hypothesis of the lemma we have that $\Gamma, \Gamma' \vdash \diamond$, so by applying ENV we conclude that $\Gamma, \Gamma' \vdash u : T$. For all the rules that have $forms(\Gamma) \models C$ as a premiss we use the idempotence property of the logical entailment. All the other cases are trivial. $\square$

We denote by $\Gamma \vdash \mathcal{J}\{M/x\}$ the capture-avoiding substitution of x by M inside an arbitrary typing judgement $\mathcal{J}$.

Definition C.4 (Substitution)

- $(\Gamma \vdash \diamond)\{M/x\} = \Gamma \vdash \diamond$

- $(\Gamma \vdash T :: k)\{M/x\} = \Gamma \vdash T\{M/x\} :: k$

- $(\Gamma \vdash T <: U)\{M/x\} = \Gamma \vdash T\{M/x\} <: U\{M/x\}$

- $(\Gamma \vdash N : T)\{M/x\} = \Gamma \vdash N\{M/x\} : T\{M/x\}$

- $(\Gamma \vdash P)\{M/x\} = \Gamma \vdash P\{M/x\}$

Note that the substitution applies to types too, but not to the typing environment.

The substitution lemma is a standard tool for proving the preservation of types at run-time. During process evaluation, variables are instantiated by terms, and the substitution lemma states that all typing judgements are preserved by the type-preserving substitution of variables.

Lemma C.5 (Substitution) *If $\Gamma, x : T \vdash \mathcal{J}$ and $\Gamma \vdash M : T$, then $\Gamma \vdash \mathcal{J}\{M/x\}$.*

Proof. We proceed by cases, depending on the judgement $\mathcal{J}$:

$\mathcal{J} = \diamond$ Trivial since $\Gamma \vdash \diamond$ by ENV-BINDING.

$\mathcal{J} = M : U$ The proof proceeds by induction on the length of the derivation of $\Gamma, x : T \vdash M : U$, and case analysis on the last applied rule. The first base case is ENV, where $\Gamma, x : T \vdash u : U$ for some u and by the premises of the rule $\Gamma, x : T \vdash \diamond$ and $u : U \in \Gamma, x : T$. If $u = x$, then we are done since $U = T$ (by ENV-BINDING), $\Gamma \vdash M : T$ (by hypothesis), and $(u : U)\{M/u\} = M : T$. If $u \neq x$, then $u : U \in \Gamma$ and $\Gamma \vdash \diamond$ (by inverting ENV-BINDING from $\Gamma, x : T \vdash \diamond$), so by ENV we have that $\Gamma \vdash u : U$, as desired. The other base case is TUPLE, where we use the substitution property of the logical entailment relation. The induction steps follow directly from the induction hypothesis.

The proof of the other judgements is simple and uses the substitution property of logical entailment for some of the base cases. $\qquad\square$

The next lemma shows that the order of the bindings and formulas inside a well-formed typing environment does not matter.

Lemma C.6 (Exchange) *If $\Gamma_1, \Gamma_2, \Gamma_3, \Gamma_4 \vdash \mathcal{J}$ and $\Gamma_1, \Gamma_3, \Gamma_2, \Gamma_4 \vdash \diamond$, then $\Gamma_1, \Gamma_3, \Gamma_2, \Gamma_4 \vdash \mathcal{J}$.*

Proof. Trivial since only the $\Gamma \vdash \diamond$ judgement depends on the order of the elements of Γ, and $\Gamma_1, \Gamma_3, \Gamma_2, \Gamma_4 \vdash \diamond$ is given explicitly as a premise. The proof is by induction on the length of the derivation of $\Gamma_1, \Gamma_2, \Gamma_3, \Gamma_4 \vdash \mathcal{J}$. $\qquad\square$

The next lemma states that well-typing is preserved by structural equivalence.

Lemma C.7 (Structural Equivalence Preserves Typing) *If $\Gamma \vdash P$ and $P \equiv Q$, then $\Gamma \vdash Q$.*

Proof. The proof is by induction on the length of the derivation of $P \equiv Q$ and case analysis on the last applied rule. The cases for $P \mid \mathbf{0} \equiv P$ (EQ-ZERO-ID), $P \mid Q \equiv Q \mid P$ (EQ-PAR-COMM), and new $a_1 : T_1.$new $a_2 : T_2.P \equiv$ new $a_2 : T_2.$new $a_1 : T_1.P$ (EQ-BIND-SWAP) are trivial.

Case (EQ-PAR-ASSOC) We assume that $\Gamma \vdash (P \mid Q) \mid R$ and we show that $\Gamma \vdash P \mid (Q \mid R)$. From the assumption, by inverting the PROC-PAR rule we obtain that there exist $\Gamma_{P|Q}$ and Γ_R such that $P \mid Q \rightsquigarrow \Gamma_{P|Q}$, and $\Gamma, \Gamma_{P|Q} \vdash R$, and $R \rightsquigarrow \Gamma_R$, and $\Gamma, \Gamma_R \vdash P \mid Q$. From $P \mid Q \rightsquigarrow \Gamma_{P|Q}$ we obtain that there exist Γ_P and Γ_Q such that $P \rightsquigarrow \Gamma_P$, $Q \rightsquigarrow \Gamma_Q$ and $\Gamma_{P|Q} = \Gamma_P, \Gamma_Q$. From $\Gamma, \Gamma_R \vdash P \mid Q$, $P \rightsquigarrow \Gamma_P$, and $Q \rightsquigarrow \Gamma_Q$ we obtain that $\Gamma, \Gamma_R, \Gamma_Q \vdash P$ and $\Gamma, \Gamma_R, \Gamma_P \vdash Q$. These can be transformed by Lemma C.6 (Exchange) to $\Gamma, \Gamma_Q, \Gamma_R \vdash P$ and $\Gamma, \Gamma_P, \Gamma_R \vdash Q$ respectively. From the latter and the previously obtained $\Gamma, \Gamma_P, \Gamma_Q \vdash R$ we obtain that $\Gamma, \Gamma_P \vdash Q \mid R$. This and $\Gamma, \Gamma_Q, \Gamma_R \vdash P$ allow us to conclude that $\Gamma \vdash P \mid (Q \mid R)$.

Case (EQ-SCOPE) In the scope extrusion case we assume that $\Gamma \vdash$ new $a : T.(P \mid Q)$ and $a \notin fn(P)$ and we show that $\Gamma \vdash P \mid$ new $a : T.Q$. From the assumption by inverting the rules PROC-NEW and PROC-PAR we obtain that there exist Γ_P and Γ_Q such that $Q \rightsquigarrow \Gamma_Q$, and $\Gamma, a : T, \Gamma_Q \vdash P$, and $P \rightsquigarrow \Gamma_P$, and $\Gamma, a : T, \Gamma_P \vdash Q$. Since $a \notin fn(P)$ we can apply Lemma C.2 (Weakening) to $\Gamma, a : T, \Gamma_Q \vdash P$ and obtain $\Gamma, \Gamma_Q \vdash P$. From $\Gamma, a : T, \Gamma_P \vdash Q$ by Lemma C.6 (Exchange) we get $\Gamma, \Gamma_P, a : T \vdash Q$, thus by PROC-NEW also $\Gamma, \Gamma_P \vdash$ new $a : T.Q$. By PROC-PAR this implies that $\Gamma \vdash P \mid$ new $a : T.Q$.

Case (Eq-Ctxt) The inductive case corresponds to the closure of structural equivalence under application of evaluation contexts. For $\mathcal{E} = \mathsf{new}\ \widetilde{a} : \widetilde{T}.[\,]\mid R$ we have that $\Gamma \vdash \mathcal{E}[P]$, $\mathcal{E}[P] \equiv \mathcal{E}[Q]$, and $P \equiv Q$. From $\Gamma \vdash \mathcal{E}[P]$ by inverting Proc-New and Proc-Par we get that there exist Γ_P and Γ_R such that $P \rightsquigarrow \Gamma_P$, and $\Gamma, \widetilde{a} : \widetilde{T}, \Gamma_P \vdash R$, and $R \rightsquigarrow \Gamma_R$, and $\Gamma, \widetilde{a} : \widetilde{T}, \Gamma_R \vdash P$. From $\Gamma, \widetilde{a} : \widetilde{T}, \Gamma_R \vdash P$ and $P \equiv Q$, by the induction hypothesis we get that $\Gamma, \widetilde{a} : \widetilde{T}, \Gamma_R \vdash Q$. In order to use Proc-Par and Proc-New to conclude that $\Gamma \vdash \mathcal{E}[Q]$ we additionally need to show that $\Gamma, \widetilde{a} : \widetilde{T}, \Gamma_Q \vdash R$. We prove this below as a separate lemma (Lemma C.8).

$\square$

Lemma C.8 (Structural Equivalence Preserves Environments) *If $P \rightsquigarrow \Gamma_P$ and $\Gamma, \Gamma_P \vdash R$ and $P \equiv Q$, then there exists Γ_Q such that $Q \rightsquigarrow \Gamma_Q$ and $\Gamma, \Gamma_Q \vdash R$.*

Proof. By induction on the length of the derivation of $P \equiv Q$. Most base cases are immediate, while for Eq-Par-Scope and Eq-Bind-Swap we need to rearrange parts of the typing environment using Lemma C.6. In all we choose Γ_Q to be the same as Γ_P up to certain rearrangements of the bindings.

The inductive case corresponds to the closure under application of evaluation contexts (rule Eq-Ctxt). Let $\mathcal{E} = \mathsf{new}\ \widetilde{a} : \widetilde{T}.\ [\,]\mid R'$ so that $P = \mathcal{E}[P']$, $Q = \mathcal{E}[Q']$ and $P' \equiv Q'$. In this case we have that $\mathsf{new}\ \widetilde{a} : \widetilde{T}.\ P' \mid R' \rightsquigarrow \Gamma_P$. If this judgement is derived without any application of Extr-Par then everything is immediate. In case Extr-Par is applied, we have that $\Gamma_P = \widetilde{a} : \widetilde{T}, \Gamma'_P, \Gamma'_R$ where $P' \rightsquigarrow \Gamma_{P'}$ and $R' \rightsquigarrow \Gamma_{R'}$. Without loss of generality we can assume that the bound names in P' and R' are disjoint so $dom(\Gamma_{P'}) \cap dom(\Gamma_{R'}) = \emptyset$ (otherwise we α-rename). From $\Gamma, \widetilde{a} : \widetilde{T}, \Gamma_{P'}, \Gamma_{R'} \vdash R$ by Lemma C.1 we obtain that $\Gamma, \widetilde{a} : \widetilde{T}, \Gamma_{P'}, \Gamma_{R'} \vdash \diamond$ and consequently that $\Gamma, \widetilde{a} : \widetilde{T}, \Gamma_{R'}, \Gamma_{P'} \vdash \diamond$. We apply Lemma C.6 (Exchange) and get that $\Gamma, \widetilde{a} : \widetilde{T}, \Gamma_{R'}, \Gamma_{P'} \vdash R$. We can therefore apply the induction hypothesis to infer that there exists $\Gamma_{Q'}$ such that $Q' \rightsquigarrow \Gamma_{Q'}$ and $\Gamma, \widetilde{a} : \widetilde{T}, \Gamma_{R'}, \Gamma_{Q'} \vdash R$. Up to further rearrangements using Lemma C.6 (Exchange) this is equivalent to $\Gamma, \Gamma_Q \vdash R$, our conclusion. $\square$

Lemma C.9 (Reduction Preserves Environments) *If $P \rightsquigarrow \Gamma_P$ and $\Gamma, \Gamma_P \vdash R$ and $P \to Q$, then there exists Γ_Q such that $Q \rightsquigarrow \Gamma_Q$ and $\Gamma, \Gamma_Q \vdash R$.*

Proof. By induction on the derivation of $P \to Q$. All base cases are trivial since then $\Gamma_P = \emptyset$. The inductive cases correspond to the closure under application of evaluation contexts (rule Red-Ctxt) and the closure under structural equivalence (rule Red-Eq).

In case Red-Ctxt is the last applied rule we know that there exists some evaluation context $\mathcal{E} = \mathsf{new}\ \widetilde{a} : \widetilde{T}.\ [\,]\mid R'$ such that $P = \mathcal{E}[P']$, $Q = \mathcal{E}[Q']$ and $P' \to Q'$. If $\mathsf{new}\ \widetilde{a} : \widetilde{T}.\ P' \mid R' \rightsquigarrow \Gamma_P$ is derived without any application of Extr-Par then the conclusion is easy to derive. If instead Extr-Par is used in the derivation we have that there exist $\Gamma_{P'}$ and $\Gamma_{R'}$ such that $P' \rightsquigarrow \Gamma_{P'}$ and $R' \rightsquigarrow \Gamma_{R'}$ and $\mathsf{new}\ \widetilde{a} : \widetilde{T}.\ P' \mid R' \rightsquigarrow \widetilde{a} : \widetilde{T}, \Gamma_{P'}, \Gamma_{R'}$. Since we know that $P' \to Q'$ and after rearrangements of $\Gamma, \widetilde{a} : \widetilde{T}, \Gamma_{P'}, \Gamma_{R'} \vdash R$ using Lemma C.6 (Exchange) we can apply the induction hypothesis to infer that there exists $\Gamma_{Q'}$ such that $Q' \rightsquigarrow \Gamma_{Q'}$ and $\Gamma, \widetilde{a} : \widetilde{T}, \Gamma_{Q'}, \Gamma_{R'} \vdash R$.

In case $P \to Q$ is derived using Red-Eq we have that $P \rightsquigarrow \Gamma_P$ and $P \equiv P'$ and $P' \to Q'$ and $Q' \equiv Q$. Since $P \rightsquigarrow \Gamma_P$ and $P \equiv P'$ and since by hypothesis $\Gamma, \Gamma_P \vdash R$ if we apply Lemma C.8 (Structural Equivalence Preserves Environments) we get that there exists $\Gamma_{P'}$ such that $P' \rightsquigarrow \Gamma_{P'}$ and $\Gamma, \Gamma_{P'} \vdash R$. Since also $P' \to Q'$ we can apply the induction hypothesis and infer that there exists $\Gamma_{Q'}$ such that $Q' \rightsquigarrow \Gamma_{Q'}$ and $\Gamma, \Gamma_{Q'} \vdash R$. Since $Q' \equiv Q$ we can apply Lemma C.8 again to obtain exactly what we had to prove: there exists Γ_Q such that $Q \rightsquigarrow \Gamma_Q$ and $\Gamma, \Gamma_Q \vdash R$. $\square$

Lemma C.10 (Sound Statement Evaluation) *If $S \Downarrow_\sharp M$ then $\models S = M$.*

Proof. The proof is by induction on the structure of the derivation of $S \Downarrow_\sharp M$. The base case EV-TERM implies that S is syntactically equal to M, so the property is trivially true. The two inductive cases are given below:

Case (EV-CONSTR) Assume that $S = f(S_1, \ldots, S_n)$ and $\forall i \in [1, n].\ S_i \Downarrow_\sharp M_i$. We can apply the induction hypothesis on the latter and obtain that $\forall i \in [1, n].\ \models S_i = M_i$. By the reflexivity of equality and the replacement property of the logic we can conclude that $\models f(S_1, \ldots, S_n) = f(M_1, \ldots, M_n)$.

Case (EV-TUPLE) Assume that $S = \langle S_1, \ldots, S_n \rangle$ and $\forall i \in [1, n].\ S_i \Downarrow_\sharp M_i$. We can apply the induction hypothesis on the latter and obtain that $\forall i \in [1, n].\ \models S_i = M_i$. By the reflexivity of equality and the replacement property of the logic we can conclude that $\models \langle S_1, \ldots, S_n \rangle = \langle M_1, \ldots, M_n \rangle$.

Case (EV-DESTR) Assume that $S = g^\sharp(S_1, \ldots, S_n)$, $\forall i \in [1, n].\ S_i \Downarrow_\sharp M_i$, and $g(M_1, \ldots, M_n) \Downarrow N$. We need to show that $\models g^\sharp(S_1, \ldots, S_n) = N$. From $S_i \Downarrow_\sharp M_i$ by the induction hypothesis it follows that $\models S_i = M_i$. From $g(M_1, \ldots, M_n) \Downarrow N$ by the assumption on destructors in Proposition A.1 we deduce that $\models g^\sharp(M_1, \ldots, M_n) = N$, and therefore by the replacement property of the logic also $\models g^\sharp(S_1, \ldots, S_n) = N$.

$\square$

Lemma C.11 (Substitution Variant) *If $\Gamma, x : T, x = N \vdash \mathcal{J}$, $\Gamma \vdash N : T$, and $x \notin fv(N)$ then $\Gamma \vdash \mathcal{J}\{N/x\}$.*

Proof. By induction on the derivation of $\Gamma, x : T, x = N \vdash \mathcal{J}$. It uses the following derived statement: if $S \cup \{x = N\} \models C$ then $S \models C\{N/x\}$, which follows from the substitution property of the logic, reflexivity of the equality relation, and idempotence. $\square$

Remark C.12 The substitution variant given in Lemma C.11 is stronger than the standard substitution lemma (Lemma C.5).

Proof. We assume that $\Gamma, x : T \vdash \mathcal{J}$ and $\Gamma \vdash M : T$. Since $\Gamma, x : T, x = M \vdash \diamond$, we can apply Lemma C.2 (Weakening) to $\Gamma, x : T \vdash \mathcal{J}$ and get that $\Gamma, x : T, x = M \vdash \mathcal{J}$. Additionally from $\Gamma, x : T \vdash \mathcal{J}$ we derive that $\Gamma, x : T \vdash \diamond$, so $x \notin dom(\Gamma)$, which by $\Gamma \vdash M : T$ means that $x \notin fn(M)$. So we can apply Lemma C.11 (Substitution Variant) to conclude that $\Gamma \vdash \mathcal{J}\{M/x\}$. $\square$

Lemma C.13 (Bound Weakening) *If $\Gamma', u : U, \Gamma'' \vdash \mathcal{J}$ and $\Gamma, u : U, \Gamma'' \vdash T <: U$ then $\Gamma', u : T, \Gamma'' \vdash \mathcal{J}$*

Proof. By induction on the derivation of $\Gamma', u : U, \Gamma'' \vdash \mathcal{J}$. $\square$

Lemma C.14 (Equivalent Types) *If $\Gamma, N = M \vdash \diamond$ and $free(T) \subseteq dom(\Gamma)$ then $\Gamma, N = M \vdash T\{N/x\} <:> T\{M/x\}$*

Proof. By induction on the structure of T. If T is a tuple type we use the expansivity and replacement properties of the logic. The other cases are trivial. $\square$

The following lemma states that the reduction rules for destructors are consistent with the typing rules.

Proposition C.15 (Typing Destructors Consistent) *If $g \neq \mathsf{ver}$, $g : (T_1, \ldots, T_n) \mapsto T$, $g(M_1, \ldots, M_n) \Downarrow N$ then if $\Gamma \vdash M_i : T_i$ for all $i \in [1, n]$ we also have that $\Gamma \vdash N : T$.*

Proof. Immediate from the rules in Table 15. □

Lemma C.16 (Inverting Tuple) *If* $\Gamma \vdash \langle \widetilde{M} \rangle : \langle \widetilde{y} : \widetilde{T} \rangle \{C\}$ *then* $\forall j \in [1, m]. \Gamma \vdash M_j : T_j$ *and* $forms(\Gamma) \models C\{\widetilde{M}/\widetilde{y}\}$.

Proof. The proof of the lemma is by induction on the derivation of $\Gamma \vdash \langle \widetilde{M} \rangle : \langle \widetilde{y} : \widetilde{T} \rangle \{C\}$. There are two cases we need to consider. If the last applied rule is TUPLE then the conclusion is immediate. On the other hand, if the last rule is SUB then we need a little more work. In this latter case we know that $\Gamma \vdash \langle \widetilde{M} \rangle : T''$ and $\Gamma \vdash T'' <: \langle \widetilde{y} : \widetilde{T} \rangle \{C\}$. We distinguish two sub-cases depending on the last applied subtyping rule in the derivation of $\Gamma \vdash T'' <: \langle \widetilde{y} : \widetilde{T} \rangle \{C\}$.

If this is SUB-TUPLE then $T'' = \langle \widetilde{y} : \widetilde{U} \rangle \{C'\}$ such that $\forall j. \Gamma \vdash U_j <: T_j$ and $forms(\Gamma) \cup \{C'\} \models C$. By the induction hypothesis we get that $\forall j. \Gamma \vdash M_j : U_j$ and $forms(\Gamma) \models C'\{\widetilde{M}/\widetilde{y}\}$. From $\Gamma \vdash M_j : U_j$ and $\Gamma \vdash U_j <: T_j$ by rule SUB we get that $\Gamma \vdash M_j : T_j$ for any j. From $forms(\Gamma) \cup \{C'\} \models C$ by substitution we obtain that $forms(\Gamma) \cup \{C'\{\widetilde{M}/\widetilde{y}\}\} \models C\{\widetilde{M}/\widetilde{y}\}$, which together with $forms(\Gamma) \models C'\{\widetilde{M}/\widetilde{y}\}$ yields by idempotence that $forms(\Gamma) \models C\{\widetilde{M}/\widetilde{y}\}$.

If the last applied rule in the derivation of $\Gamma \vdash T'' <: \langle \widetilde{y} : \widetilde{T} \rangle \{C\}$ is SUB-PUB-TNT then we know that $\Gamma \vdash T'' :: \mathsf{pub}$ and $\Gamma \vdash \langle \widetilde{y} : \widetilde{T} \rangle \{C\} :: \mathsf{tnt}$. From the latter by inverting KIND-TUPLE-TNT we obtain that $\forall j. \Gamma \vdash T_j :: \mathsf{tnt}$ and $forms(\Gamma) \models C$, and by substitution $forms(\Gamma) \models C\{\widetilde{M}/\widetilde{y}\}$. From $\Gamma \vdash \langle \widetilde{M} \rangle : T''$ and $\Gamma \vdash T'' :: \mathsf{pub}$ we can prove by induction that there exists a type $T''' = \langle \widetilde{T'''} \rangle \{C'''\}$ such that $\Gamma \vdash \langle \widetilde{M} \rangle : T'''$ and $\forall j. \Gamma \vdash T_j''' :: \mathsf{pub}$. By rule TUPLE we have that $\forall j. \Gamma \vdash M_j : T_j'''$ so from SUB-PUB-TNT we obtain that $\forall j. \Gamma \vdash M_j : T_j$. This concludes the proof of the lemma. □

Definition C.17 (Conservative Environment Extension) *Let* Γ, Γ' *be typing environments and* σ *be a substitution. We say that* Γ' *is a conservative extension of* Γ *with respect to* σ, *written* $\Gamma \prec_\sigma \Gamma'$ *if the following conditions hold:*

1. $\Gamma \vdash \diamond$ *and* $\Gamma' \vdash \diamond$;

2. $dom(\Gamma') = dom(\Gamma) \cup \{x_1, \ldots, x_n, y_1, \ldots, y_m\}$;

3. $\forall i \in [1, n]. \Gamma \vdash x_i\sigma : \Gamma'(x_i)$;

4. $\forall j \in [1, m]. \Gamma \vdash y_j\sigma : \Gamma'(y_j)$;

5. $\forall C. forms(\Gamma') \models C \Rightarrow forms(\Gamma) \models C\sigma$.

Lemma C.18 (Correctness of Statement Compilation) *Let* Γ *be a typing environment and* $\sigma = \{N_1/x_1, \ldots, N_n/x_n, M_1/y_1, \ldots, M_m/y_m\}$ *be a substitution. Let* C *be a formula such that* $free(C) \subseteq \{\widetilde{x}, \widetilde{y}\}$, *and* $T_{l+1}, \ldots, T_m$ *be types such that if there exist a type* T *and a term* $N \in \{N_1, \ldots, N_n, M_1, \ldots, M_m\}$ *such that* $\Gamma \vdash N : T$ *and* $\Gamma \nvdash T :: \mathsf{pub}$ *and* $\Gamma \nvdash T :: \mathsf{tnt}$ *then* $\forall j \in [l+1, m]. \Gamma \vdash M_j : T_j$ *and* $forms(\Gamma) \models C\sigma$. *For every statement* S *such that* $free(S) \subseteq \{\widetilde{x}, \widetilde{y}\}$ *and for every typing environment* Γ' *such that* $\Gamma \prec_\sigma \Gamma'$, *if* $[|S|]_{\Gamma'}^{y_{l+1}:T_{l+1}, \ldots, y_m:T_m; C} = \Gamma''$ *and* $S\sigma \Downarrow_\sharp \mathsf{true}$, *then we have that* $\Gamma \prec_\sigma \Gamma''$.

Proof. We assume the premises of the lemma and prove by induction on the structure of S that for every Γ' such that $\Gamma \prec_\sigma \Gamma'$, assuming that $[|S|]_{\Gamma'} = \Gamma''$ and $S\sigma \Downarrow_\sharp \mathsf{true}$, we have $\Gamma \prec_\sigma \Gamma''$. We distinguish several cases corresponding to the ones in the statement compilation function.

Case $[|S|]_{\Gamma'} = \Gamma', S$. Since $S\sigma \Downarrow_\sharp \mathsf{true}$ by Lemma C.10 we obtain that $\models S\sigma = \mathsf{true}$, which according to our assumptions on the logic implies that $\models S\sigma$. Having this we can easily show that $\Gamma \prec_\sigma \Gamma', S$.

Case $[|S_1 \wedge^\sharp S_2|]_{\Gamma'} = binds(\Gamma_{21}) \sqcap binds(\Gamma_{12}), (\bigwedge_{C \in forms(\Gamma_{21}) \cup forms(\Gamma_{12})} C)$ where $\Gamma_{21} = [|S_1|]_{[|S_2|]_{\Gamma'}}$ and $\Gamma_{12} = [|S_2|]_{[|S_1|]_{\Gamma'}}$. Since $S_1\sigma \wedge^\sharp S_2\sigma \Downarrow_\sharp$ true by the semantics of $\wedge$ we have that $S_1\sigma \Downarrow_\sharp$ true and $S_2\sigma \Downarrow_\sharp$ true. By applying the induction hypothesis to $S_1\sigma$ and to $S_2\sigma$ we obtain that $\Gamma \prec_\sigma [|S_1|]_{\Gamma'}$ and $\Gamma \prec_\sigma [|S_2|]_{\Gamma'}$. By choosing Γ' to be $[|S_1|]_{\Gamma'}$ in the induction hypothesis, and since $S_2\sigma \Downarrow_\sharp$ true we obtain that $\Gamma \prec_\sigma [|S_2|]_{[|S_1|]_{\Gamma'}}$, i.e., $\Gamma \prec_\sigma \Gamma_{21}$. Similarly, we infer that $\Gamma \prec_\sigma \Gamma_{12}$. The rest is straightforward in this case.

Case $[|S_1 \vee^\sharp S_2|]_\Gamma = binds([|S_1|]_{\Gamma'}) \sqcup binds([|S_2|]_{\Gamma'}), ((\bigwedge_{C \in forms([|S_1|]_{\Gamma'})} C) \vee (\bigwedge_{C \in forms([|S_2|]_{\Gamma'})} C))$. From $S_1\sigma \vee^\sharp S_2\sigma \Downarrow_\sharp$ true we infer that $S_1\sigma \Downarrow_\sharp$ true or $S_2\sigma \Downarrow_\sharp$ true. By the induction hypothesis it follows that $\Gamma \prec_\sigma [|S_1|]_{\Gamma'}$ or $\Gamma \prec_\sigma [|S_2|]_{\Gamma'}$. We can show by unfolding the definitions that if $\Gamma \prec_\sigma \Gamma_1$ and $\Gamma_2 \vdash \diamond$ or if $\Gamma \prec_\sigma \Gamma_2$ and $\Gamma_1 \vdash \diamond$ and if $dom(\Gamma_1) = dom(\Gamma_2)$ then $\Gamma \prec_\sigma binds(\Gamma_1) \sqcup binds(\Gamma_2)$. In our case this means that $\Gamma \prec_\sigma binds([|S_1|]_{\Gamma'}) \sqcup binds([|S_2|]_{\Gamma'})$. Additionally we need to show that $forms(\Gamma) \models ((\bigwedge_{C \in forms([|S_1|]_{\Gamma'})} C\sigma) \vee (\bigwedge_{C \in forms([|S_2|]_{\Gamma'})} C\sigma))$. This is immediate since we already know that $\forall C \in forms([|S_1|]_{\Gamma'}).\ forms(\Gamma) \models C\sigma$ or $\forall C \in forms([|S_2|]_{\Gamma'}).\ forms(\Gamma) \models C\sigma$.

Case $[|\mathsf{check}^\sharp(v_M, v_K) = v_N|]_{\Gamma'} = \Gamma'[v_N : T^*], \mathsf{check}^\sharp(v_M, v_K) = v_N$ where $v_M, v_K, v_N \in \{\widetilde{x}, \widetilde{y}\}$, $\Gamma'(v_K) = \mathsf{VerKey}(T^*)$ and $\Gamma', \mathsf{false} \nvdash T^* :: \mathsf{tnt}$ and $\Gamma' \vdash T^* :: \mathsf{pub}$. If $\Gamma' = \Gamma_1, v_N : U^*, \Gamma_2$ then $\Gamma'[v_N : T^*] = \Gamma_1, v_N : T^*, \Gamma_2$, so we need to show that $\Gamma \vdash v_N\sigma : T^*$.

Since $\mathsf{check}^\sharp(v_M\sigma, v_K\sigma) = v_N\sigma \Downarrow_\sharp$ true by the semantics of the check destructor we get that there exist terms N and K such that $v_M\sigma = \mathsf{sign}(N, K)$, $v_K\sigma = \mathsf{vk}(K)$, $v_N\sigma = N$ and $\mathsf{check}(\mathsf{sign}(N, K), \mathsf{vk}(K)) \Downarrow N$. Since $\Gamma \prec_\sigma \Gamma'$ and $\Gamma'(v_K) = \mathsf{VerKey}(T^*)$ it follows that $\Gamma \vdash \mathsf{vk}(K) : \mathsf{VerKey}(T^*)$. From this and $\Gamma \nvdash T^* :: \mathsf{tnt}$ (we get this from $\Gamma', \mathsf{false} \nvdash T^* :: \mathsf{tnt}$ by the contrapositive of Lemma C.2 (Weakening)) we can prove by induction that there exists T' such that $\Gamma \vdash T' < T^*$ and $\Gamma \vdash \mathsf{vk}(K) : \mathsf{VerKey}(T')$ and $\Gamma \vdash K : \mathsf{SigKey}(T')$. Because $\Gamma \prec_\sigma \Gamma'$ we additionally get that $\Gamma \vdash \mathsf{sign}(N, K) : \Gamma'(v_M)$, where the precise type given to $\mathsf{sign}(N, K)$ is not important. We can show by induction on $\Gamma \vdash K : \mathsf{SigKey}(T')$ that these imply $\Gamma \vdash \mathsf{sign}(N, K) : \mathsf{Signed}(T')$. Additionally we already know that $\Gamma \vdash K : \mathsf{SigKey}(T')$ so by Proposition C.15 it follows that $\Gamma \vdash N : T'$, and, by subsumption $\Gamma \vdash N : T^*$, as desired.

Case $[|\mathsf{check}^\sharp(v_M, v_K) = v_N|]_{\Gamma'} = \Gamma'[v_N : T^*, y_{l+1} : T_{l+1}, \ldots, y_m : T_m], C, \mathsf{check}^\sharp(v_M, v_K) = v_N$ where $\Gamma'(v_K) = \mathsf{VerKey}(T^*)$, $\Gamma', \mathsf{false} \nvdash T^* :: \mathsf{tnt}$ and $\Gamma', \mathsf{false} \nvdash T^* :: \mathsf{pub}$. The proof proceeds similarly to the previous case. We additionally need to show that $\forall j \in [l+1, m].\ \Gamma \vdash M_j : T_j$ and $forms(\Gamma) \models C\sigma$, but this is immediate from one of the conditions in the hypothesis since $v_N\sigma \in \{\widetilde{N}, \widetilde{M}\}$, $\Gamma \nvdash T^* :: \mathsf{pub}$, $\Gamma \nvdash T^* :: \mathsf{tnt}$, and, like in the previous case, $\Gamma \vdash v_N\sigma : T^*$.

Case $[|\mathsf{dec}^\sharp(v_M, v_K) = v_N|]_{\Gamma'} = \Gamma'[v_N : T^*, y_{l+1} : T_{l+1}, \ldots, y_m : T_m], C, \mathsf{dec}^\sharp(v_M, v_K) = v_N$ where $\Gamma'(v_M) = \mathsf{PubEnc}(T^*)$, $\Gamma', \mathsf{false} \nvdash T^* :: \mathsf{tnt}$, and $\Gamma', \mathsf{false} \nvdash T^* :: \mathsf{pub}$. The proof proceeds similarly to the previous two cases. We will only show that $\Gamma \vdash v_N\sigma : T^*$.

From $\mathsf{dec}^\sharp(v_M\sigma, v_K\sigma) = v_N\sigma$ by the semantics of dec there exist N and K such that $v_M\sigma = \mathsf{enc}(N, \mathsf{pk}(K))$, $v_K\sigma = K$, $v_N\sigma = N$ and $\mathsf{dec}(\mathsf{enc}(N, \mathsf{pk}(K)), K) \Downarrow N$. Since $\Gamma \prec_\sigma \Gamma'$ and $\Gamma'(v_M) = \mathsf{PubEnc}(T^*)$ we obtain that $\Gamma \vdash \mathsf{enc}(N, \mathsf{pk}(K)) : \mathsf{PubEnc}(T^*)$. Since $\Gamma \nvdash T^* :: \mathsf{tnt}$ we can prove by induction on $\Gamma \vdash \mathsf{enc}(N, \mathsf{pk}(K)) : \mathsf{PubEnc}(T^*)$ that $\Gamma \vdash N : T^*$.

Case $[|v_M = \mathsf{hash}(v_N)|]_{\Gamma'} = \Gamma'[v_N : T^*, y_{l+1} : T_{l+1}, \ldots, y_m : T_m], C, v_M = \mathsf{hash}(v_N)$ where $\Gamma'(v_M) = \mathsf{Hash}(T^*)$, $\Gamma', \mathsf{false} \nvdash T^* :: \mathsf{tnt}$, and $\Gamma', \mathsf{false} \nvdash T^* :: \mathsf{pub}$. The proof is the very similar to the previous two cases.

Case $[|\langle v_{M_1}, \ldots, v_{M_k} \rangle = v_N|] = \Gamma'[v_{M_1} : U_1, \ldots, v_{M_k} : U_k], C^*\{\widetilde{v_M}/\widetilde{z}\}, \langle v_{M_1}, \ldots, v_{M_k} \rangle = v_N$ where $\Gamma'(v_N) = \langle z_1 : U_1, \ldots, z_k : U_k \rangle\{C^*\}$. We have to show that $\forall i.\ \Gamma \vdash v_{M_i} : U_i$ and $forms(\Gamma) \models C^*\{\widetilde{v_M}\sigma/\widetilde{z}\}$.

By the operational semantics of $=$ we get that $v_N\sigma = \langle v_{M_1}\sigma, \ldots, v_{M_k}\sigma\rangle$. Since $\Gamma \prec_\sigma \Gamma'$ we have that $\Gamma \vdash \langle v_{M_1}\sigma, \ldots, v_{M_k}\sigma\rangle : \langle z_1 : U_1, \ldots, z_k : U_k\rangle\{C^*\}$. From Lemma C.16 (Inverting Tuple) we can conclude that $\forall i. \ \Gamma \vdash v_{M_i} : U_i$ and $\mathit{forms}(\Gamma) \models C^*\{\widetilde{v_M}\sigma/\widetilde{z}\}$.

$\square$

The following lemma will allow us to apply Lemma C.18 in the subject-reduction proof.

Lemma C.19 *If* $\Gamma(s_{n,m,S}) = \mathsf{Stm}(\langle \widetilde{y} : \widetilde{T}\rangle\{\exists\widetilde{x}. \ C\})$ *and* $\Gamma \vdash \mathsf{zk}(\widetilde{N}, \widetilde{M}) : T$ *and* $\exists T' \ \exists N \in \{\widetilde{N}, \widetilde{M}\}$ *such that* $\Gamma \vdash N : T'$ *and* $\Gamma \nvdash T' :: \mathsf{pub}$ *and* $\Gamma \nvdash T' :: \mathsf{tnt}$ *then we can conclude that* $\forall j \in [l+1, m]. \ \Gamma \vdash M_j : T_j$ *and* $\mathit{forms}(\Gamma) \models C\{\widetilde{N}/\widetilde{x}\}\{\widetilde{M}/\widetilde{y}\}$.

Proof. By induction on $\Gamma \vdash \mathsf{zk}(\widetilde{N}, \widetilde{M}) : T$. We distinguish three cases corresponding to the last rule applied in the derivation:

Case (SUB) We know that there exists T'' such that $\Gamma \vdash \mathsf{zk}(\widetilde{N}, \widetilde{M}) : T''$ so we can apply the induction hypothesis and directly obtain the conclusion (the conclusion of the lemma only depends on the zero-knowledge term being well typed, not on its precise type.).

Case (ZK-UN) We have that $\forall i \in [1, n]. \ \Gamma \vdash N_i : \mathsf{Un}$ and $\forall j \in [1, m]. \ \Gamma \vdash M_i : \mathsf{Un}$. Hence there exists no term in $\{\widetilde{N}, \widetilde{M}\}$ which we can type to a non-public and non-tainted type. As a consequence, the lemma is vacuously satisfied since the hypothesis does not hold.

Case (ZK) Since $\Gamma(s_{n,m,S}) = \mathsf{Stm}(\langle \widetilde{y} : \widetilde{T}\rangle\{\exists\widetilde{x}. \ C\})$ we get that $\Gamma \vdash \langle \widetilde{M}\rangle : \langle \widetilde{y} : \widetilde{T}\rangle\{C\{\widetilde{N}/\widetilde{x}\}\}$. We use the general result about inverting the TUPLE rule from Lemma C.16.

$\square$

Theorem C.20 (Subject-reduction) *If* $\Gamma \vdash P$ *and* $P \rightarrow Q$ *then* $\Gamma \vdash Q$.

Proof. The proof is by induction on the derivation of $P \rightarrow Q$ and case analysis on the last applied rule.

Case (RED-I/O) Assume that $\mathsf{out}(a, M).P \mid \mathsf{in}(a, x).Q \rightarrow P \mid Q\{M/x\}$ and $\Gamma \vdash \mathsf{out}(a, M).P \mid \mathsf{in}(a, x).Q$. By PROC-PAR we obtain that $\Gamma \vdash \mathsf{out}(a, M).P$ and $\Gamma \vdash \mathsf{in}(a, x).Q$. The former by PROC-OUT implies that $\Gamma \vdash a : \mathsf{Ch}(T)$, $\Gamma \vdash M : T$ and $\Gamma \vdash P$, while from the latter by PROC-IN we additionally infer that $\Gamma, x : T \vdash Q$. From Lemma C.5 (Substitution) it follows that $\Gamma \vdash Q\{M/x\}$. As a consequence of Lemma C.2 (Weakening) and the rule PROC-PAR we obtain that $\Gamma \vdash P \mid Q\{M/x\}$.

Case (RED-!I/O) This case is very similar to the previous one.

Case (RED-DESTR) Let $\mathsf{let} \ x = g(\widetilde{M}) \ \mathsf{then} \ P \ \mathsf{else} \ Q \rightarrow P\{N/x\}$, $g(\widetilde{M}) \Downarrow N$ and $\Gamma \vdash \mathsf{let} \ x = g(\widetilde{M}) \ \mathsf{then} \ P \ \mathsf{else} \ Q$. In case $g \neq \mathsf{ver}$ by PROC-DES we get that $g : (T_1, \ldots, T_n) \mapsto T$, $\forall i \in [1, n]. \ \Gamma \vdash M_i : T_i$, and $\Gamma, x : T, x = g^\sharp(M_1, \ldots, M_n) \vdash P$. By Lemma C.15 we infer that $\Gamma \vdash N : T$. This allows us to apply Lemma C.2 (Weakening) to get that $\Gamma, x : T, x = g^\sharp(M_1, \ldots, M_n), x = N \vdash P$. Because of our assumption that $\models g^\sharp(M_1, \ldots, M_n) = N$ we can apply strengthening to obtain $\Gamma, x : T, x = N \vdash P$. By Lemma C.11 (Substitution Variant) we can finally infer that $\Gamma \vdash P\{N/x\}$.

In case $g = \mathsf{ver}$ we have that $\mathsf{let} \ x = \mathsf{ver}_{n,m,l,S}(\mathsf{zk}_{n,m,S}(\widetilde{N}, \widetilde{M}), M_1, \ldots, M_l) \ \mathsf{then} \ P \ \mathsf{else} \ Q \rightarrow P\{\langle M_{l+1}, \ldots, M_m\rangle/x\}$ where $\mathsf{ver}_{n,m,l,S}(\mathsf{zk}_{n,m,S}(\widetilde{N}, \widetilde{M}), M_1, \ldots, M_l) \Downarrow \langle M_{l+1}, \ldots, M_m\rangle$, so by the semantics of the ver destructor we have $S'\sigma \Downarrow_\sharp \mathsf{true}$ where $\sigma = \{\widetilde{N}/\widetilde{x}\}\{\widetilde{M}/\widetilde{y}\}$ and $S' = $

$S\{\widetilde{x}/\widetilde{\alpha}\}\{\widetilde{y}/\widetilde{\beta}\}$. We distinguish two cases depending on the last applied rule in the derivation of $\Gamma \vdash \mathsf{let}\ x = \mathsf{ver}_{n,m,l,S}(\mathsf{zk}_{n,m,S}(\widetilde{N}, \widetilde{M}), M_1, \ldots, M_l)\ \mathsf{then}\ P\ \mathsf{else}\ Q$. If the zero-knowledge proof is verified by an opponent that is type-checked using rule Proc-Ver-Un then $\Gamma(s_{n,m,S}^{un}) = \mathsf{Stm}(\mathsf{Un})$ and $\Gamma \vdash \mathsf{zk}_{n,m,S}(\widetilde{N}, \widetilde{M}) : \mathsf{Un}$. No matter whether the latter is derived using rule Zk or Zk-Un we have that $\forall j \in [1,m].\ \Gamma \vdash M_i : \mathsf{Un}$. By rule Tuple we obtain that $\Gamma \vdash \langle M_{l+1}, \ldots, M_m \rangle : \langle \widetilde{\mathsf{Un}} \rangle \{\mathsf{true}\}$, which by Sub-Pub-Tnt and Sub yields $\Gamma \vdash \langle M_{l+1}, \ldots, M_m \rangle : \mathsf{Un}$. Since we also know that $\Gamma, x : \mathsf{Un} \vdash P$ we can apply the substitution lemma and get that $\Gamma \vdash P\{\langle M_{l+1}, \ldots, M_m \rangle / x\}$, which concludes this case.

If the zero-knowledge proof is verified by a honest participant that is type-checked using rule Proc-Ver, then $\Gamma(s_{n,m,S}) = \mathsf{Stm}(T)$, where $T = \langle \widetilde{y} : \widetilde{T} \rangle \{\exists \widetilde{x}.\ C\}$, $\Gamma \vdash \mathsf{zk}_{n,m,S}(\widetilde{N}, \widetilde{M}) :$ $\mathsf{ZKProof}_{n,m,S}(T')$, where $T' = \langle \widetilde{y} : \widetilde{U} \rangle \{\mathsf{true}\}$, and

$$\forall j \in [1,l].\ \Gamma \vdash M_j : T_j \tag{1}$$

$$[|S'|]_{\Gamma'}^{y_{l+1}:T_{l+1},\ldots,y_m:T_m;C} = \Gamma'', \text{ for } \Gamma' = \Gamma, \widetilde{x} : \widetilde{\top}, y_1 : T_1, \ldots, y_l : T_l, y_{l+1} : U_{l+1}, \ldots, y_m : U_m \tag{2}$$

$$forms(\Gamma'') \models C \tag{3}$$

$$\forall j \in [l+1,m].\ \Gamma'' \vdash y_j : T_j \tag{4}$$

$$\Gamma, x : \langle y_{l+1} : T_{l+1}, \ldots, y_m : T_m \rangle \{\exists \widetilde{x}.\ C\{M_i/y_i\}_{i \in [1,l]}\} \vdash P \tag{5}$$

We need to show that $\Gamma \vdash P\{\langle M_{l+1}, \ldots, M_m \rangle / x\}$. From (5) by Lemma C.5 (Substitution) it suffices to show that $\Gamma \vdash \langle M_{l+1}, \ldots, M_m \rangle : \langle y_{l+1} : T_{l+1}, \ldots, y_m : T_m \rangle \{\exists \widetilde{x}.\ C\{M_i/y_i\}_{i \in [1,l]}\}$. By rule Tuple it suffices to show that $\forall j \in [l+1, m].\ \Gamma \vdash M_j : T_j$ and that $forms(\Gamma) \models \exists \widetilde{x}.\ C\{\widetilde{M}/\widetilde{y}\}$. We will show these conditions using Lemma C.18 (Correctness of Statement Compilation).

In order to apply Lemma C.18 we need to show that $\Gamma \prec_\sigma \Gamma'$, which holds as a consequence of (1) and $forms(\Gamma) = forms(\Gamma')$. By Lemma C.19 we obtain that if $\exists T' \exists N \in \{\widetilde{N}, \widetilde{M}\}$ such that $\Gamma \vdash N : T'$ and $\Gamma \nvdash T' :: \mathsf{pub}$ and $\Gamma \nvdash T' :: \mathsf{tnt}$ then $\forall j \in [l+1, m].\ \Gamma \vdash M_j : T_j$ and $forms(\Gamma) \models C\sigma$. Additionally, we have that $[|S'|]_{\Gamma'}^{y_{l+1}:T_{l+1},\ldots,y_m:T_m;C} = \Gamma''$ from (2) and that $S'\sigma \Downarrow_\sharp \mathsf{true}$. We can therefore apply Lemma C.18 and obtain that $\Gamma \prec_\sigma \Gamma''$. In particular this means that $\forall j \in [1,m].\ \Gamma \vdash M_j : \Gamma''(y_j)$ and $\forall C.\ forms(\Gamma'') \models C \Rightarrow forms(\Gamma) \models C\sigma$.

Let $j \in [l+1, m]$. From (4) we know that $\Gamma'' \vdash y_j : T_j$, so there exists T'_j such that $\Gamma''(y_j) = T'_j$ and $\Gamma \vdash T'_j <: T_j$. So $\Gamma \vdash M_j : \Gamma''(y_j)$ is actually $\Gamma \vdash M_j : T'_j$ and by Sub we infer that $\forall j \in [l+1, m].\ \Gamma \vdash M_j : T_j$.

From (3) we have that $forms(\Gamma'') \models C$, so from what we obtained from Lemma C.18 we infer that $forms(\Gamma) \models C\sigma$. By the property of existential quantification we assume of the authorization logic we obtain that $forms(\Gamma) \models \exists \widetilde{x}.C\{\widetilde{M}/\widetilde{y}\}$, which concludes the proof of this case.

Case (Red-Else) Immediate from Proc-Des, Proc-Ver or Proc-Ver-Un.

Case (Red-Split) Let $\Gamma \vdash \mathsf{let}\ \langle \widetilde{x} \rangle = \langle \widetilde{M} \rangle\ \mathsf{in}\ P$ and $\mathsf{let}\ \langle \widetilde{x} \rangle = \langle \widetilde{M} \rangle\ \mathsf{in}\ P \rightarrow P\{\widetilde{M}/\widetilde{x}\}$. By Proc-Split we get that $\Gamma \vdash \langle \widetilde{M} \rangle : \langle \widetilde{y} : \widetilde{T} \rangle \{C\}$ and that $\Gamma, \widetilde{x} : \widetilde{T}, \langle \widetilde{x} \rangle = \langle \widetilde{M} \rangle, C\{\widetilde{x}/\widetilde{y}\} \vdash P$. From the former, by rule Tuple we get that $\forall i.\ \Gamma \vdash M_i : T_i$ and $forms(\Gamma) \models C\{\widetilde{M}/\widetilde{y}\}$. This gives us by monotonicity, expansivity and replacement that $forms(\Gamma) \cup \{\langle \widetilde{x} \rangle = \langle \widetilde{M} \rangle\} \cup \{x_i = M_i \mid i \in [1,n]\} \models C\{\widetilde{x}/\widetilde{y}\}$. From the latter, by the equality of tuples and Lemma C.2 (Weakening) we obtain that $\Gamma, \widetilde{x} : \widetilde{T}, \langle \widetilde{x} \rangle = \langle \widetilde{M} \rangle, x_1 = M_1, \ldots, x_n = M_n, C\{\widetilde{x}/\widetilde{y}\} \vdash P$. Putting these two together, by Lemma C.3 (Strengthening) we infer that $\Gamma, \widetilde{x} : \widetilde{T}, x_1 = M_1, \ldots, x_n = M_n \vdash P$. Finally, by repeatedly applying Lemmas C.6 and C.11 (Substitution Variant) we conclude that $\Gamma \vdash P\{\widetilde{M}/\widetilde{x}\}$.

Case RED-CTXT Let $\mathcal{E} = \mathsf{new}\ \widetilde{a} : \widetilde{T}.([\ ]\ |\ R)$ and assume that $\Gamma \vdash \mathcal{E}[P]$ and $P \to Q$. From $\Gamma \vdash \mathcal{E}[P]$ by reverting rule PROC-NEW we get that $\Gamma, \widetilde{a} : \widetilde{T} \vdash P\ |\ R$. From this by reverting rule PROC-PAR we infer that there exist Γ_R and Γ_P such that $R \rightsquigarrow \Gamma_R$ and $\Gamma, \widetilde{a} : \widetilde{T}, \Gamma_R \vdash P$ and $P \rightsquigarrow \Gamma_P$ and $\Gamma, \widetilde{a} : \widetilde{T}, \Gamma_P \vdash R$. Since $\Gamma, \widetilde{a} : \widetilde{T}, \Gamma_R \vdash P$ and $P \to Q$ by the induction hypothesis we obtain that $\Gamma, \widetilde{a} : \widetilde{T}, \Gamma_R \vdash Q$. By Lemma C.9 (Reduction Preserves Environments) since $P \rightsquigarrow \Gamma_P$ and $\Gamma, \widetilde{a} : \widetilde{T}, \Gamma_P \vdash R$ and $P \to Q$ we infer that there exists Γ_Q such that $Q \rightsquigarrow \Gamma_Q$ and $\Gamma, \widetilde{a} : \widetilde{T}, \Gamma_Q \vdash R$. By rule PROC-PAR it follows that $\Gamma, \widetilde{a} : \widetilde{T} \vdash Q\ |\ R$, and by PROC-NEW we conclude that $\Gamma \vdash \mathcal{E}[Q]$.

Case RED-EQ Assume that $\Gamma \vdash P$ and $P \equiv P'$ and $P' \to Q'$ and $Q' \equiv Q$. From $\Gamma \vdash P$ and $P \equiv P'$ by Lemma C.7 (Structural Equivalence Preserves Typing) we obtain that $\Gamma \vdash P'$. By the induction hypothesis this yields $\Gamma \vdash Q'$. By applying Lemma C.7 again we conclude that $\Gamma \vdash Q$.

$\square$

Lemma C.21 (Correct Environment Extraction) *If $Q \rightsquigarrow \Gamma_Q$ then there exists an evaluation context $\mathcal{E} = \mathsf{new}\ \widetilde{b} : \widetilde{U}.\ [\ ]\ |\ Q'$ such that $dom(\Gamma_Q) = \widetilde{b}$, $Q \equiv \mathcal{E}[\mathsf{assume}\ C_1\ |\ \dots\ |\ \mathsf{assume}\ C_n]$, and $forms(\Gamma_Q) = \{C_1, \dots, C_n\}$.*

Proof. We prove this by induction on the derivation of $Q \rightsquigarrow \Gamma_Q$. The cases in which $\Gamma_Q = \emptyset$ are all trivial. If $\mathsf{assume}\ C \rightsquigarrow C$ then we take $\mathcal{E} = [\ ]\ |\ \mathbf{0}$ and all conditions are immediate.

If $\mathsf{new}\ a : T.P \rightsquigarrow a : T, \Gamma_P$ where $P \rightsquigarrow \Gamma_P$. We can apply the induction hypothesis for $P \rightsquigarrow \Gamma_P$ and obtain that there exists $\mathcal{E} = \mathsf{new}\ \widetilde{b} : \widetilde{U}.[\ ]\ |\ P'$ such that $dom(\Gamma_P) = \widetilde{b}$, $P \equiv \mathcal{E}[\mathsf{assume}\ C_1\ |\ \dots\ |\ \mathsf{assume}\ C_n]$, and $forms(\Gamma_P) = \{C_1, \dots, C_n\}$. We can take $\mathcal{E}' = \mathsf{new}\ a : T.\mathcal{E}$ which satisfies all required conditions.

If $P\ |\ Q \rightsquigarrow \Gamma_P, \Gamma_Q$ where $P \rightsquigarrow \Gamma_P$, $Q \rightsquigarrow \Gamma_Q$. From $P \rightsquigarrow \Gamma_P$ by the induction hypothesis we obtain that there exists $\mathcal{E}_P$ such that $\mathcal{E}_P = \mathsf{new}\ \widetilde{b^P} : \widetilde{U^P}.[\ ]\ |\ P'$ such that $dom(\Gamma_P) = \widetilde{b^P}$, $P \equiv \mathcal{E}_P[\mathsf{assume}\ C_1^P\ |\ \dots\ |\ \mathsf{assume}\ C_n^P]$, and $forms(\Gamma_P) = \{C_1^P, \dots, C_n^P\}$. Similarly, from $Q \rightsquigarrow \Gamma_Q$ we obtain that $\mathcal{E}_Q = \mathsf{new}\ \widetilde{b^Q} : \widetilde{U^Q}.[\ ]\ |\ Q'$ such that $dom(\Gamma_Q) = \widetilde{b^Q}$, $Q \equiv \mathcal{E}_Q[\mathsf{assume}\ C_1^Q\ |\ \dots\ |\ \mathsf{assume}\ C_m^Q]$, and $forms(\Gamma_Q) = \{C_1^Q, \dots, C_m^Q\}$. We consider the context $\mathcal{E} = \mathcal{E}^P[\mathcal{E}^Q]$. First, note that this context has the required shape $\mathcal{E} = \mathsf{new}\ \widetilde{b^P} : \widetilde{U^P}.\mathsf{new}\ \widetilde{b^Q} : \widetilde{U^Q}.[\ ]\ |\ P'\ |\ Q'$. Then, since $dom(\Gamma_P) = \widetilde{b^P}$ and $dom(\Gamma_Q) = \widetilde{b^Q}$ we obtain that $dom(\Gamma_P, \Gamma_Q) = \widetilde{b^P}, \widetilde{b^Q}$. Since $P \equiv \mathcal{E}_P[\mathsf{assume}\ C_1^P\ |\ \dots\ |\ \mathsf{assume}\ C_n^P]$ and $Q \equiv \mathcal{E}_Q[\mathsf{assume}\ C_1^Q\ |\ \dots\ |\ \mathsf{assume}\ C_m^Q]$ by EQ-CTXT we have that $P\ |\ Q \equiv \mathcal{E}[C_1^P\ |\ \dots\ |\ \mathsf{assume}\ C_n^P\ |\ \mathsf{assume}\ C_1^Q\ |\ \dots\ |\ \mathsf{assume}\ C_m^Q]$. Finally, since $forms(\Gamma_P) = \{C_1^P, \dots, C_n^P\}$ and $forms(\Gamma_Q) = \{C_1^Q, \dots, C_m^Q\}$ we infer that that $forms(\Gamma_P\ |\ \Gamma_Q) = \{C_1^P, \dots, C_n^P, C_1^Q, \dots, C_m^Q\}$, which concludes the proof of this case and of the lemma. $\square$

The next theorem states that the type system enforces safety.

Theorem C.22 (Safety) *For every closed process P and every Γ, if $\Gamma \vdash P$ then P is safe.*

Proof. We assume that $P \to^* \mathsf{new}\ \widetilde{a} : \widetilde{T}.(\mathsf{assert}\ C\ |\ Q)$. From Theorem C.20 (Subject-reduction) it follows that $\Gamma \vdash \mathsf{new}\ \widetilde{a} : \widetilde{T}.(\mathsf{assert}\ C\ |\ Q)$, where $fn(P) \subseteq dom(\Gamma)$. By PROC-NEW we have that $\Gamma, \widetilde{a} : \widetilde{T} \vdash \mathsf{assert}\ C\ |\ Q$. We chose Γ_Q such that $Q \rightsquigarrow \Gamma_Q$ and $dom(\Gamma_Q) \cap (dom(\Gamma) \cup \widetilde{a}) = \emptyset^5$.

By Lemma C.21 we obtain that there exists $\mathcal{E} = \mathsf{new}\ \widetilde{b} : \widetilde{U}.[\ ]\ |\ Q'$ such that $dom(\Gamma_Q) = \widetilde{b}$, $Q \equiv \mathcal{E}[\mathsf{assume}\ C_1\ |\ \dots\ |\ \mathsf{assume}\ C_n]$, and $forms(\Gamma_Q) = \{C_1, \dots, C_n\}$.

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[5] This second condition can always be satisfied by α-renaming the bound names in Q.

From $dom(\Gamma_Q) \cap (dom(\Gamma) \cup \widetilde{a}) = \emptyset$ and $\Gamma, \widetilde{a} : \widetilde{T} \vdash \diamond$ (Lemma C.1) we have that that $\Gamma, \widetilde{a} : \widetilde{T}, \Gamma_Q \vdash \diamond$. By inverting PROC-PAR we obtain that $\Gamma, \widetilde{a} : \widetilde{T}, \Gamma_Q \vdash$ assert C, so by inverting PROC-ASSERT we obtain that $forms(\Gamma_Q) \models C$, which is equivalent to $\{C_1, \dots, C_n\} \models C$.

Finally, we need to show that $fn(C) \cap dom(\Gamma_Q) = \emptyset$. This follows from $fn(C) \subseteq dom(\Gamma)$ and $dom(\Gamma_Q) \cap \widetilde{c} = \emptyset$. $\qquad\square$

Lemma C.23 (Public Tainted)

1. $\Gamma \vdash T :: \mathsf{pub}$ *if and only if* $\Gamma \vdash T <: \mathsf{Un}$;

2. $\Gamma \vdash T :: \mathsf{tnt}$ *if and only if* $\Gamma \vdash \mathsf{Un} <: T$.

Proof. The forward implications are immediate by SUB-PUB-TNT and KIND-UN. For the reverse implication of *1.* the only rule that can derive $\Gamma \vdash T <: \mathsf{Un}$ is SUB-PUB-TNT, thus we have that $\Gamma \vdash T :: \mathsf{pub}$. Analogously, $\Gamma \vdash \mathsf{Un} <: T$ can only be derived using SUB-PUB-TNT and therefore $\Gamma \vdash T :: \mathsf{tnt}$. $\qquad\square$

Lemma C.24 (Public Down - Tainted Up) *Let* $\Gamma \vdash T <: T'$ *then*

1. *if* $\Gamma \vdash T' :: \mathsf{pub}$ *then* $\Gamma \vdash T :: \mathsf{pub}$;

2. *if* $\Gamma \vdash T :: \mathsf{tnt}$ *then* $\Gamma \vdash T' :: \mathsf{tnt}$.

Proof. We prove *1* and *2* together by induction on the derivation of $\Gamma \vdash T <: T'$. The SUB-TUPLE case uses the idempotence of the logic to show *2*. The other cases are immediate from the rules in Table 13. $\qquad\square$

Lemma C.25 (Subtyping Preorder)

Reflexivity: *If* $\Gamma \vdash \diamond$ *and* $free(T) \subseteq dom(\Gamma)$ *then* $\Gamma \vdash T <: T$;

Transitivity: *If* $\Gamma \vdash T_1 <: T_2$ *and* $\Gamma \vdash T_2 <: T_3$ *then* $\Gamma \vdash T_1 <: T_3$.

Proof. Reflexivity follows directly from the SUB-REFL rule. For transitivity we use Lemma C.24 (Public Down - Tainted Up) if any of the subtyping judgements in the premise are derived by SUB-PUB-TNT. In case the subtyping judgements in the premise are derived using SUB-TUPLE we apply Lemma C.13 (Bound Weakening), and use the monotonicity and idempotence properties of the logic. $\qquad\square$

Lemma C.26 (Universal Type) *If* $\Gamma \vdash \diamond$ *then* $\Gamma \vdash \mathsf{Un} <:> T$ *for all*

$$T \in \{\mathsf{Ch}(\mathsf{Un}), \langle x_1 : \mathsf{Un}, \dots, x_n : \mathsf{Un} \rangle \{\mathsf{true}\}, \mathsf{SigKey}(\mathsf{Un}), \mathsf{VerKey}(\mathsf{Un}), \mathsf{Signed}(\mathsf{Un}),$$
$$\mathsf{PubKey}(\mathsf{Un}), \mathsf{PrivKey}(\mathsf{Un}), \mathsf{PubEnc}(\mathsf{Un}), \mathsf{Hash}(\mathsf{Un}), \mathsf{ZKProof}_{n,m,S}(\mathsf{Un})\}$$

Proof. It is immediate from Table 12 that $\Gamma \vdash T :: \mathsf{pub}$ and $\Gamma \vdash T :: \mathsf{tnt}$ for all T's above, thus by Lemma C.23 it follows that $\Gamma \vdash \mathsf{Un} <:> T$. $\qquad\square$

Proposition C.27 (Constructors Compatible with Type Un) *For all constructors f we have that* $f : (\mathsf{Un}, \dots, \mathsf{Un}) \mapsto \mathsf{Un}$.

Lemma C.28 (Term Un-typability) *For all M we have that* $\emptyset \vdash_{\mathsf{Un}} M : \mathsf{Un}$.

Proof. By induction on the structure of M and using Lemma C.26 (Universal Type). $\qquad\square$

Proposition C.29 (Destructors Compatible with Type Un) *For all destructors $g \neq \mathsf{ver}$ we have that* $g : (\mathsf{Un}, \dots, \mathsf{Un}) \mapsto \mathsf{Un}$.

Lemma C.30 (Opponent Typability) *For all opponents O we have that $\Gamma^{env} \vdash_{\mathsf{Un}} O$, for $\Gamma^{env} = s^{un}_{n_1,m_1,S_1} : \mathsf{Stm}(\mathsf{Un}), \ldots, s^{un}_{n_k,m_k,S_k} : \mathsf{Stm}(\mathsf{Un})$ where $S_1, \ldots, S_k$ are all the statements used in O.*

Proof. By induction on the structure of O and using Lemmas C.13 (Bound Weakening), C.26 (Universal Type), and C.28 (Term Un-typability). $\qquad\square$

Theorem C.31 (Robust Safety) *Let $\Gamma = s_{n_1,m_1,S_1} : \mathsf{Stm}(T_1), \ldots, s_{n_k,m_k,S_k} : \mathsf{Stm}(T_k)$. For every closed process P, if $\Gamma \vdash_{\mathsf{Un}} P$ then P is robustly safe.*

Proof. The statement follows from Theorem C.22 (Safety), Lemma C.30 (Opponent Typability), Lemma C.2 (Weakening), and rule PROC-PAR. $\qquad\square$

D   Blind Signatures and Secret Hashes

DAA relies on blind signatures, blinded messages, and secret hashes that are modeled in our calculus by the new constructors bsign, blind, and $\mathsf{hash}_{\mathsf{Private}}$, respectively. The unblinding of blind signatures is ruled by the $\mathsf{unblind}$ destructor $(\mathsf{unblind}(\mathsf{bsign}(\langle \mathsf{blind}(x,y) \rangle, z), y, \mathsf{bvk}(z)) \Downarrow \mathsf{sign}(\langle x \rangle, z))$, while their verification is denoted by the bcheck destructor $(\mathsf{bcheck}(\mathsf{sign}(x,z), \mathsf{bvk}(z)) \Downarrow x)$, respectively. Each blind signature carries a logical formula, which is transferred to the signature obtained after unblinding.

We extend our type system with the following types: $\mathsf{Blind}(T)$ describes blinded messages of type T, $\mathsf{BlindSigKey}(T)$ and $\mathsf{BlindVerKey}(T)$ describe signing and verification keys for blind signatures of messages of type T, $\mathsf{BlindSigned}(T)$ describes blind signatures of messages of type T, $\mathsf{Blinder}(T)$ describes a blinding factor for messages of type T, and $\mathsf{Hash}_{\mathsf{Private}}(T)$ describes secret hashes of messages of type T. Note that $\mathsf{Blinder}(T)$ and $\mathsf{BlindSigKey}(T)$ are generative, i.e., can be used as the type of names generated with the *new* construct. The typing rules for constructors are defined as expected:

$$\mathsf{blind} : (T, \mathsf{Blinder}(T)) \mapsto \mathsf{Blind}(T)$$
$$\mathsf{bvk} : (\mathsf{BlindSigKey}(T)) \mapsto \mathsf{BlindVerKey}(T)$$
$$\mathsf{bsign} : (\langle x : \mathsf{Blind}(T) \rangle \{C\}, \mathsf{BlindSigKey}(\langle x : \mathsf{Blind}(T) \rangle \{C\})) \mapsto \mathsf{BlindSigned}(\langle x : \mathsf{Blind}(T) \rangle \{C\})$$
$$\mathsf{hash}_{\mathsf{Private}} : (T) \mapsto \mathsf{Hash}_{\mathsf{Private}}(T)$$

The typing rules for destructors are reported below:

$$\mathsf{unblind} : (\mathsf{BlindSigned}(\langle x : \mathsf{Blind}(T) \rangle \{C\}), \mathsf{Blinder}(T), \mathsf{BlindVerKey}(\langle x : \mathsf{Blind}(T) \rangle \{C\})) \mapsto \mathsf{Signed}(T)$$
$$\mathsf{bcheck} : (\mathsf{Signed}(T), \mathsf{BlindVerKey}(\langle z : \mathsf{Blind}(T) \rangle \{C\})) \mapsto \langle x : T \rangle \{\exists y.C\{\mathsf{blind}(x,y)/z\}\}$$

The evaluation of the destructor $\mathsf{unblind}$ yields a signature of type $\mathsf{Signed}(T)$, where T is determined by the type $\mathsf{Blinder}(T)$ of the blinding factor. Notice that the evaluation rule checks that the blinding factor matches the one in the blind signature. The evaluation of the destructor bcheck yields the content of the signature, whose type $\langle x : T \rangle \{\exists y.C\{\mathsf{blind}(x,y)/z\}\}$ is determined by the type $\mathsf{BlindVerKey}(\langle z : \mathsf{Blind}(T) \rangle \{C\})$ of the verification key. Notice that the variable z in C refers to the blinded message and is thus replaced by $\mathsf{blind}(x,y)$, where the blinding factor y is unknown and thus existentially quantified. Finally, the compilation of the statements concerning

blind signatures is defined similarly to the statements for regular signatures:

$$[|\mathsf{bcheck}(v_M, v_K)\ \mathsf{eq}^\sharp\ v_N|]_{\Gamma,C} \;=\; \Gamma[v_N : \langle x : T\rangle\{\exists y.C'\{\mathsf{blind}(x,y)/z\}], \mathsf{bcheck}^\sharp(v_M, v_K)\ \mathsf{eq}^\sharp\ v_N$$

$$\text{if } \Gamma(v_K) = \mathsf{BlindVerKey}(\langle z : \mathsf{Blind}(T)\rangle\{C'\}) \text{ and } \Gamma \nvdash T : \mathsf{tnt}\ \wedge\ \Gamma \vdash T : \mathsf{pub}$$

$$[|\mathsf{bcheck}(v_M, v_K)\ \mathsf{eq}^\sharp\ v_N|]_{\Gamma,C} \;=\; \Gamma[v_N : \langle x : T\rangle\{\exists y.C'\{\mathsf{blind}(x,y)/z\}, y_{l+1} : T_{l+1}, \ldots, T_m : T_m],$$
$$C, \mathsf{bcheck}^\sharp(v_M, v_K)\ \mathsf{eq}^\sharp\ v_N$$
$$\text{if } \Gamma(v_K) = \mathsf{BlindVerKey}(\langle z : \mathsf{Blind}(T)\rangle\{C'\}) \text{ and } \Gamma \nvdash T : \mathsf{tnt}\ \wedge\ \Gamma \nvdash T : \mathsf{pub}$$

The kinding rules are straightforward and follow the same pattern as the rules for signatures:

KIND-BLIND
$$\frac{\Gamma \vdash \diamond \qquad free(T) \subseteq dom(\Gamma)}{\Gamma \vdash \mathsf{Blind}(T) :: k}$$

KIND-BLINDER
$$\frac{\Gamma \vdash T :: \mathsf{pub} \qquad \Gamma \vdash T :: \mathsf{tnt}}{\Gamma \vdash \mathsf{Blinder}(T) :: k}$$

KIND-BLINDSIGNED
$$\frac{\Gamma \vdash \diamond \qquad free(T) \subseteq dom(\Gamma)}{\Gamma \vdash \mathsf{BlindSigned}(T) :: k}$$

KIND-BLINDVERKEY-PUB
$$\frac{\Gamma \vdash \diamond \qquad free(T) \subseteq dom(\Gamma)}{\Gamma \vdash \mathsf{BlindVerKey}(T) :: \mathsf{pub}}$$

KIND-BLINDVERKEY-TNT
$$\frac{\Gamma \vdash T :: \mathsf{tnt}}{\Gamma \vdash \mathsf{BlindVerKey}(T) :: \mathsf{tnt}}$$

KIND-BLINDSIGKEY
$$\frac{\Gamma \vdash T :: \mathsf{tnt}}{\Gamma \vdash \mathsf{BlindSigKey}(T) :: k}$$

KIND-HASHPRIVATE
$$\frac{\Gamma \vdash T :: \mathsf{pub} \qquad \Gamma \vdash T :: \mathsf{tnt}}{\Gamma \vdash \mathsf{Hash}_{\mathsf{Private}}(T) :: k}$$

The same holds for the subtyping rules:

SUB-BLIND-INV
$$\frac{\Gamma \vdash T <:> U}{\Gamma \vdash \mathsf{Blind}(T) <: \mathsf{Blind}(U)}$$

SUB-BLINDER-INV
$$\frac{\Gamma \vdash T <:> U}{\Gamma \vdash \mathsf{Blinder}(T) <: \mathsf{Blinder}(U)}$$

SUB-BLINDSIGNED-INV
$$\frac{\Gamma \vdash T <:> U}{\Gamma \vdash \mathsf{BlindSigned}(T) <: \mathsf{BlindSigned}(U)}$$

SUB-BLINDSIGKEY-INV
$$\frac{\Gamma \vdash T <:> U}{\Gamma \vdash \mathsf{BlindSigKey}(T) <: \mathsf{BlindSigKey}(U)}$$

SUB-BLINDVERKEY-COV
$$\frac{\Gamma \vdash T <: U}{\Gamma \vdash \mathsf{BlindVerKey}(T) <: \mathsf{BlindVerKey}(U)}$$

SUB-HASH-PRIV-INV
$$\frac{\Gamma \vdash T <:> U}{\Gamma \vdash \mathsf{Hash}_{\mathsf{Private}}(T) <: \mathsf{Hash}_{\mathsf{Private}}(U)}$$